On self-complementary chordal graphs

by

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Certificate

It is certified that the work contained in the thesis entitled "*On self-complementary chordal graphs*" by *K.Balaji* has been carried out under my supervision and guidance and that this work has not been submitted elsewhere for any degree or diploma.

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K. Balaji
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K.Balaji

Dedicated in memory
of my father
Shri. P.S.V.Kannan

Synopsis

Perfect Graphs and several classes of perfect graphs had been studied because of the nice combinatorial structure and interesting applications [24], [101] and [179].

The four problems, namely,

(i) finding the clique number of a graph

(ii) finding the chromatic number of a graph

(iii) finding the stability number of a graph

(iv) finding the clique covering number of a graph

which are NP-hard in general [91] can be solved in polynomial time when restricted to perfect graphs [105]. This result has intensified the algorithmic interest in perfect graphs. In 1962 Berge [14] posed the following conjecture (see also [24] and [101]).

Conjecture 1 : *For a graph G the following conditions are equivalent.*

(i) *G is α-perfect.*

(ii) *G is χ-perfect.*

(iii) *G has no induced subgraph isomorphic to C_{2k+1} or $\bar{C}_{2k+1}$ for $k \geq 2$.*

Lovasz [133] established that a graph is χ-perfect if and only if it is α-perfect. This is called the Perfect Graph Theorem. As a consequence of the Perfect Graph Theorem, Conjecture 1 reduces to Conjecture 2.

Conjecture 2 : *A graph is a perfect graph (χ-perfect or α-perfect) if and only if it has no induced subgraph isomorphic to C_{2k+1} or $\bar{C}_{2k+1}$ for all $k \geq 2$.*

Conjecture 2 is called the Strong Perfect Graph Conjecture (SPGC) and it is still unsettled. A class of graphs is said to be complete for a conjecture if the truth of the conjecture on this class implies the truth of the conjecture in general. Corneil [70] has identified self-complementary graphs, regular graphs and various other classes of graphs to

be complete classes for SPGC. This result of Corneil motivates the study of the various classes of self-complementary perfect graphs. In this thesis we study the self-complementary chordal graphs, a class of self-complementary perfect graphs.

In Chapter 1 entitled 'Introduction' we discuss briefly about the various results on s.c. graphs and chordal graphs available in the literature.

In Chapter 2 entitled 'On the characterisations for self-complementary graphs to be chordal' we obtain the following characterisations.

(i) Let G be a self-complementary (s.c.) graph. Then G is chordal if and only if $\omega(G) = 2n$ when $p = 4n$ and $\omega(G) = 2n + 1$ when $p = 4n + 1$ where p denotes the number of vertices of G and $\omega(G)$ denotes the clique number of G.

(ii) Let G be a s.c. graph. Then G is chordal if and only if $\alpha(G) = 2n$ when $p = 4n$ and $\alpha(G) = 2n + 1$ when $p = 4n + 1$ where $\alpha(G)$ denotes the stability number of G.

(iii) Let G be a s.c. graph. Then G is chordal if and only if G has no induced subgraph isomorphic to C_4 when $p = 4n$ and G has no induced subgraph isomorphic to C_4 or C_5 when $p = 4n + 1$ where C_n denotes the cycle with n vertices.

(iv) Let G be a s.c. graph with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_p$. Then G is chordal if and only if $\sum_{i=1}^{2n} d_i = 6n^2 - 2n$ when $p = 4n$ and $\sum_{i=1}^{2n} d_i = 6n^2$ when $p = 4n + 1$.

The results (i) and (ii) have been published in reputed journals [204] and [205].

In Chapter 3 entitled 'On the existence and the construction of self-complementary chordal graphs' we prove that s.c. chordal graphs with p vertices exist if and only if $p = 4n$ or $p = 4n + 1$ for some positive integer n. We also give algorithms to construct all s.c. chordal graphs with $4n$ and $4n + 1$ vertices for all positive integers n. Though these algorithms construct all s.c. chordal graphs with $4n$ and $4n + 1$, vertices they may construct the same graph (with different labellings) repeatedly. So the isomorphism problem of s.c. chordal graphs plays a major role in the catalogue compilation of s.c. chordal graphs (list of non-isomorphic s.c. chordal graphs).

In Chapter 4 entitled 'On the isomorphism and the catalogue compilation of self-complementary chordal graphs' we prove that the isomorphism of s.c. chordal graphs, the

isomorphism of s.c. chordal graphs with 4n vertices and the isomorphism of s.c. chordal graphs with 4n+1 vertices are polynomially equivalent. Alter [5], Faradzhev [78], Kropar et. al. [127], Morris [143] and Venkatachalam [225] had compiled the catalogue of s.c. graphs with a few vertices (upto 12 vertices). From these we recognise those graphs which are chordal by using the recognition algorithms given in this chapter and obtain the catalogue of s.c. chordal graphs with atmost 12 vertices. We also obtain the catalogue of s.c. chordal graphs with 13 vertices by giving a method for obtaining all non-isomorphic s.c. chordal graphs with 4n+1 vertices from the set of all non-isomorphic s.c. chordal graphs with 4n vertices.

The results in Chapters 3 and 4 have been submitted for publication [206] and [207].

Graph parameters had been studied because of the usefulness in determining the structure of the graph. In Chapter 5 entitled 'Self-complementary chordal graphs and some graph parameters' we study the following graph parameters for s.c. chordal graphs.

(a) Chromatic number.

(b) Chromatic index.

(c) Domination number.

(d) Spectrum.

In Section 5.2 of Chapter 5 we prove that the chromatic number of s.c. chordal graphs with 4n vertices is 2n and that of s.c. chordal graphs with 4n+1 vertices is 2n+1. We also get bounds on the chromatic number of s.c. perfect graphs. We prove that the upper bounds are attained if and only if the graph is s.c. chordal.

In Section 5.3 of Chapter 5 we give a sufficient condition for a s.c. chordal graph with 4n vertices to be a Class 1 graph (a graph whose chromatic index is equal to its maximum degree) and establish the Class 1 property for some classes of s.c. chordal graphs with 4n vertices. We also obtain bounds for the chromatic index of s.c. chordal graphs.

In Section 5.4 of Chapter 5 we prove that the domination number and the independent domination number can be atmost 2n for s.c. chordal graphs with 4n vertices and atmost 2n+1 for s.c. chordal graphs with 4n+1 vertices.

In Section 5.5 of Chapter 5 we prove that the least positive integer for which there exist cospectral s.c. chordal graphs is 12. We also obtain bounds for the maximum eigen-value of a s.c. chordal graph.

In Chapter 6 entitled 'Conclusion' we offer some research problems on s.c. chordal graphs.

List of Publications

(i) M.R.Sridharan and K.Balaji, 'On self-complementary chordal graphs', National Academy Science Letters 20 (1997).

(ii) M.R.Sridharan and K.Balaji, 'Characterisation of self-complementary chordal graphs', accepted for publication in Discrete Mathematics.

(iii) M.R.Sridharan and K.Balaji, 'On construction of self-complementary chordal graphs', submitted for publication.

(iv) M.R.Sridharan and K.Balaji, 'On isomorphism of self-complementary chordal graphs', submitted for publication.

Contents

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Chapter 1

Introduction

1.1 General introduction

Perfect Graphs and several classes of perfect graphs had been studied because of the nice combinatorial structure and interesting applications [24], [52], [77], [101], [102], [179], [220] and [221].

The four problems, namely,

(i) finding the clique number of a graph

(ii) finding the chromatic number of a graph

(iii) finding the stability number of a graph

(iv) finding the clique covering number of a graph

which are NP-hard in general [91] can be solved in polynomial time when restricted to perfect graphs [105]. This result has intensified the algorithmic interest in perfect graphs. Soon after the introduction of perfect graphs many started to identify these graphs. Berge [16] showed that many familiar classes of graphs such as chordal graphs (also known as triangulated graphs in the literature), comparability graphs, interval graphs, unimodular graphs and line graphs of bipartite graphs are perfect. Foldes and Hammer [85] established that split graphs belong to the class of chordal graphs. Thus these graphs are also perfect. The idea of splittance of a graph namely, the minimum number of edges to be added or deleted in

order to produce a split graph was introduced by Hammer and Simone [109]. According to this definition split graphs are those graphs whose splittance is zero. Interval graphs (a class of perfect graphs) became popular because of its applications in traffic light phasing [179], seriation [101] (that is an attempt to place a set of items in their proper chronological order) and certain other optimization problems [101] and [179]. Hajos [107] first posed the problem of characterising interval graphs. The famed molecular biologist Benzer [11] was also trying to find the answer to a related problem in his investigations of the fine structure of the gene. The first characterisation of interval graphs appeared in 1962 by Lekkerker and Boland [130] followed by Gilmore and Hoffmann [99]. Fulkerson and Gross [89] gave still another characterisation. Interval graphs are perfect because they are chordal [99]. Closely related to interval graphs are circular-arc graphs. Every interval graph is a circular-arc graph but the converse is not true. Circular-arc graphs are not perfect in general. For example C_{2k+1} for all $k \geq 2$ belong to this class and are not perfect. We refer to [120], [137], [210], [211] and [223] for applications of these graphs. Pneuli et. al. [155] proved that a graph G is a permutation graph if and only if G and $\bar{G}$ are comparability graphs. This implies that permutation graphs are perfect. Isomorphism of permutation graphs has been studied in [64]. For other results on perfect graphs and their applications we refer to [13], [15], [17], [18], [19], [20], [21], [22], [23], [25], [29], [48], [49], [50], [53], [198], [230], [100], [101], [110], [118], [119], [132], [135], [138], [148], [157], [169], [170], [176], [179], [194], [195], [212], [219] and [224].

Motivated by Shannon's work [195] in 1962 Berge [14] posed the following conjecture (see also [24] and [101]).

Conjecture 1.1 : *For a graph G the following conditions are equivalent.*

(i) *G is α-perfect.*

(ii) *G is χ-perfect.*

(iii) *G has no induced subgraph isomorphic to C_{2k+1} or $\bar{C}_{2k+1}$ for $k \geq 2$.*

During the initial stages the concepts of α-perfectness and χ-perfectness were thought to be distinct and a perfect graph was defined to be a graph which is both χ-perfect and α-

perfect. Lovasz [133] established that a graph is χ-perfect if and only if it is α-perfect. This is called the Perfect Graph Theorem. Lovasz in [132] gave another equivalent condition for α-perfectness and χ-perfectness. Golumbic in [101] observed that the Perfect Graph Theorem had been almost proved by Fulkerson. As a consequence of the Perfect Graph Theorem, Conjecture1.1 reduces to Conjecture 1.2.

Conjecture 1.2 : *A graph is a perfect graph (χ-perfect or α-perfect) if and only if it has no induced subgraph isomorphic to C_{2k+1} or $\bar{C}_{2k+1}$ for all $k \geq 2$.*

Conjecture1.2 is called the Strong Perfect Graph Conjecture (SPGC) and it is still unsettled. A class of graphs is said to be valid for a conjecture if the conjecture is true for this restricted class of graphs. Many special classes of graphs called $K_{1,3}$-free graphs [152], toroidal graphs [104], $(K_4 - e)$-free graphs [153], planar graphs [221] and many other classes of graphs [42], [230], [129], [136], [146] and [222] were shown to be valid classes for SPGC. A class of graphs is said to be complete for a conjecture if the truth of the conjecture on this class implies the truth of the conjecture in general. Corneil [70] has identified self-complementary graphs, regular graphs and various other classes of graphs to be complete classes for SPGC. This result of Corneil motivates the study of various classes of self-complementary perfect graphs and self-complementary imperfect graphs. In this thesis we study the self-complementary chordal graphs, a class of self-complementary perfect graphs. For various other results on SPGC we refer to [37], [139], [147], [170], [208] and [220].

The class of self-complementary chordal graphs enjoy both the properties of self-complement graphs and the properties of chordal graphs. The class of self-complementary graphs and the class of chordal graphs had been widely studied in the literature. We discuss briefly about the various results available in these two classes.

(i) Self-complementary graphs (s.c. graphs) : The existence problem of s.c. graphs was solved independently by Ringel [178] and Sachs [190]. They prove that s.c. graphs with p vertices exist if and only if p=4n or p=4n+1 for some positive integer n. Ringel [178] obtained algorithms to construct s.c. graphs with 4n and 4n+1 vertices. Algorithms for constructing s.c. graphs with 4n and 4n+1 vertices were also obtained by Gibbs [98] by

modifying Ringel's algorithms. Harary [111] posed the problem of counting non-isomorphic s.c. graphs given the number of vertices. A complete solution for this problem was given by Read [171] and [172]. His method is based on enumertion theory originated by Redfield [175] and Polya [156] developed further by DeBruijn [35] and [36] and Harary et. al. [113]. The number of s.c. graphs (non-isomorphic) s_p for a given number of vertices p is given in the following table for $p \leq 17$.

p	1	4	5	8	9	12	13	16	17
s_p	1	1	2	10	36	720	5600	703760	11220000

An asymptotic formula for s_p as $p \to \infty$ was derived by Palmer [149]; see also Robinson [180], Sridharan [203] and Schwenk [192]. Further results concerning enumeration of s.c. graphs can be found in [3], [115], [149], [154], [172], [173], [202], [203] and [209]. The problem of deciding whether two given graphs are isomorphic or not is called the isomorphism problem. Isomorphism of s.c. graphs and regular s.c. graphs was discussed by Colbourn et. al. [65] and [66]. They prove that the isomorphism of these classes is polynomially equivalent to the general graph isomorphism. The problem of deciding whether a given graph is s.c. or not is called the recognition problem of s.c. graphs. Colbourn et. al. [65] prove that the recognition of s.c. graphs is polynomially equivalent to the general graph isomorphism. Catalogue of s.c. graphs with small number of vertices was compiled by Alter [5], Faradzhev [78], Kropar et. al. [127], Morris [143] and Venkatachalam [225]. Clapham and Kleitman [61] had obtained a necessary and sufficient condition for a degree sequence to be the degree sequence of a s.c. graph. Further results on the degree sequence of s.c. graphs are obtained in [46], [57], [73] and [74]. The existence of paths or cycles of prescribed length in s.c. graphs was investigated by Camion [41], Clapham [55] [56] and [58] and Rao [164] [165] [166] and [167]. In Camion [41] and Clapham [55] and [58] it is proved that each s.c. graph has a Hamiltonian path. S.c. graphs with atmost 5 vertices have an odd number of Hamiltonian paths and s.c. graphs with atleast 6 vertices have an even number of Hamiltonian paths [41] and [166]. According to Rao [163] each s.c. graph with p vertices $p > 5$ has a cycle of length i for each $2 \leq i \leq p-2$ but need not have a cycle of length $p-1$ or p. However if it does have

a Hamiltonian cycle then it contains cycles of all lengths $2 \leq i \leq p$ (pancyclic) [163]. The problem of the existence of Hamiltonian cycle in s.c. graphs was solved by Rao [165]. Results on the number of triangles in s.c. graphs was given by Clapham [54], Radhakrishnan Nair et. al. [160] and Rao [167]. Chao et. al. [45] studied the s.c. graphs with a given chromatic number. They prove that for each n the maximum number of vertices of any $n-chromatic$ s.c. graph is n^2. Further, for $n \geq 3$ they constructed n-chromatic s.c. graphs of diameter 2 and 3. Shannon capacity of s.c. graphs was investigated by Lovasz [134]. Various other results on s.c. graphs can be found in [2], [6], [8], [34], [51], [47], [56], [59], [60], [71], [88], [114], [140], [161], [162], [168], [187], [188], [189], [191], [213], [235] and [236].

(ii) Chordal graphs : Chordal graphs are also known as triangulated graphs, rigid circuit graphs, monotone transitive graphs and perfect elimination graphs in the literature. This class of perfect graphs has been extensively studied in the literature [12], [24], [39], [43], [75], [76], [80], [93], [94], [97], [101], [102], [106], [121], [177], [179], [181], [182], [183], [196], [199], [200] and [218]. Chordal graphs were introduced by Hajnal and Suranyi [106]. They prove that these graphs are α-perfect. Berge [12] proved the χ-perfectness of these graphs. These graphs find applications in evolutionary trees [39], archaeology [38], facility location [44], scheduling [150] and solutions of sparse systems of linear equations [182]. Many graph problems including the four classical optimization problems that are NP-hard for general graphs can be solved in polynomial time in chordal graphs [91] though testing Hamiltonicity [67], determining the domination number [31] and other problems [91] and [123] are NP-Complete for this class too. A concept called Perfect Elimination Ordering (PEO) is important in chordal graphs. It turns out that all the existing chordal graph recognition algorithms and many optimization problems including the four classical ones in chordal graphs make use of PEO [91], [101], [131], [181], [184], [197], [215] and [216]. Fulkerson and Gross [89] characterised these graphs as the graphs having a perfect elimination scheme. Dirac [75] shows these graphs as the graphs for which every minimal vertex separator induces a complete subgraph. Buneman [39], Gavril [95] and Walter [229] prove that a graph is chordal if and only if it is an intersection graph of the subtrees of a tree. Various other characterisations of chordal graphs are given in [10], [24] and [101]. Various subclasses of chordal graphs namely, split graphs, strongly chordal graphs, threshold graphs, interval graphs and k-trees had been

studied in [7], [9], [11], [26], [27], [30], [32], [40], [50], [80], [84] [85], [86], [87], [89], [99], [102], [109], [110], [126], [130], [158], [159] [183], [201] and [231]. Chordal graphs are discussed in [96], [124], [128], [142], [144], [185], [186], [214], [218], [228], [229], [233] and [234].

1.2 Outline of the thesis

In this thesis we study the class of s.c. chordal graphs which is a subclass of the class of s.c. perfect graphs.

In Chapter 2 entitled 'On the characterisations for self-complementary graphs to be chordal' we obtain the following characterisations.

(i) Let G be a s.c. graph. Then G is chordal if and only if $\omega(G) = 2n$ when $p = 4n$ and $\omega(G) = 2n+1$ when $p = 4n+1$ where p denotes the number of vertices of G and $\omega(G)$ denotes the clique number of G.

(ii) Let G be a s.c. graph. Then G is chordal if and only if $\alpha(G) = 2n$ when $p = 4n$ and $\alpha(G) = 2n+1$ when $p = 4n+1$ where $\alpha(G)$ denotes the stability number of G.

(iii) Let G be a s.c. graph. Then G is chordal if and only if G has no induced subgraph isomorphic to C_4 when $p = 4n$ and G has no induced subgraph isomorphic to C_4 or C_5 when $p = 4n+1$ where C_n denotes the cycle with n vertices.

(iv) Let G be a s.c. graph with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_p$. Then G is chordal if and only if $\sum_{i=1}^{2n} d_i = 6n^2 - 2n$ when $p = 4n$ and $\sum_{i=1}^{2n} d_i = 6n^2$ when $p = 4n+1$.

Characterisations (i) and (ii) had been published in reputed journals [204] and [205].

In Chapter 3 entitled 'On the existence and the construction of self-complementary chordal graphs' we prove that s.c. chordal graphs with p vertices exist if and only if $p = 4n$ or $p = 4n+1$ for some positive integer n. We also give algorithms to construct all s.c. chordal graphs with $4n$ and $4n+1$ vertices for any positive integer n. Though these algorithms construct all s.c. chordal graphs with $4n$ and $4n+1$ vertices, they may construct the same graph (with different labellings) repetitively. So the isomorphism problem of s.c. chordal

graphs plays a major role in the catalogue compilation of s.c. chordal graphs (list of non-isomorphic s.c. chordal graphs).

Isomorphism problem is still unsolved inspite of the several attempts made by many researchers to solve it [33] [63] [65] [66] [68] [69] [92] [123] and [174]. Isomorphism of s.c. chordal graphs apart from playing an important role in the catalogue compilation of s.c. chordal graphs seems to have close relationship with the solvability of the isomorphism problem. It appears that the isomorphism problem is solvable if and only if it is when restricted to s.c. chordal graphs. In Chapter 4 entitled 'On the isomorphism and the catalogue compilation of self-complementary chordal graphs' we prove that the isomorphism of s.c. chordal graphs, the isomorphism of s.c. chordal graphs with 4n vertices and the isomorphism of s.c. chordal graphs with 4n+1 vertices are polynomially equivalent. Alter [5], Faradzhev [78], Kropar et. al. [127], Morris [143] and Venkatachalam [225] had compiled a catalogue of s.c. graphs with a few vertices (upto 12 vertices). From this catalogue we recognise those graphs which are chordal by using the recognition algorithms given in this chapter and obtain the catalogue of s.c. chordal graphs with atmost 12 vertices. We also obtain the catalogue of s.c. chordal graphs with 13 vertices by giving a method for obtaining all non-isomorphic s.c. chordal graphs with 4n+1 vertices from the set of all non-isomorphic s.c. chordal graphs with 4n vertices.

The results in Chapters 3 and 4 have been submitted for publication [206] and [207].

In Chapter 5 entitled 'Self-complementary chordal graphs and some graph parameters' we study the following graph parameters for s.c. chordal graphs.

(a) Chromatic number.

(b) Chromatic index.

(c) Domination number.

(d) Spectrum.

In Section 5.2 we study the chromatic number of s.c. chordal graphs. We prove that the chromatic number of s.c. chordal graphs with 4n vertices is 2n and that of s.c. chordal graphs with 4n+1 vertices is 2n+1. We also obtain bounds for the chromatic number of s.c.

perfect graphs and prove that the upper bounds are attained if and only if the graph is s.c. chordal. In Section 5.3 of Chapter 5 we study the chromatic index of s.c. chordal graphs. We give a sufficient condition for a s.c. chordal graph with 4n vertices to be a Class 1 graph and establish the Class 1 property for some classes of s.c. chordal graphs with 4n vertices. We also obtain bounds for the chromatic index of s.c. chordal graphs. In Section 5.4 we study the domination number of s.c. chordal graphs. In this we prove that the domination number and the independent domination number can be atmost 2n for s.c. chordal graphs with 4n vertices and atmost 2n+1 for s.c. chordal graphs with 4n+1 vertices. In Section 5.5 we discuss the spectrum of s.c. chordal graphs. In this we prove that the least positive integer for which there exist cospectral s.c. chordal graphs (s.c. chordal graphs with same spectrum) is 12. We also obtain bounds for the maximum eigenvalue of a s.c. chordal graph.

In Chapter 6 entitled 'Conclusion' we offer some research problems on s.c. chordal graphs.

1.3 Basic definitions and notations

In this thesis $\Box$ denotes the end of a proof. Let r be a real number. The greatest integer less than or equal to r and the smallest integer greater than or equal to r are denoted by $\lfloor r \rfloor$ and $\lceil r \rceil$ respectively. Let m, n and b be non-negative integers such that $n < b$. Then $m \equiv n \pmod{b}$ if $m = bj + n$ for some non-negative integer j. The sum of a sequence of n integers $a_1, a_2, \ldots, a_n$ is denoted by $\sum_{i=1}^{n} a_i$. The empty set is denoted by $\emptyset$. Let X be a set. The number of elements in X, an element x belongs to X and an element x does not belong to X are denoted by $|X|$, $x \in X$ and $x \notin X$ respectively. Let $x \in X$. The set $X - \{x\}$ is the set obtained from X by removing the element x. The union and the intersection of two sets X and Y are denoted by $X \cup Y$ and $X \cap Y$ respectively. A set Y that is contained in a set X is denoted by $Y \subseteq X$. Let $Y \subseteq X$. We denote $x_i \in Y$ implies $x_{i+1} \in Y$ for all $1 \leq i \leq n-1$ as $x_1 \Rightarrow x_2 \Rightarrow \cdots \Rightarrow x_n \in Y$.

For the basic definitions and the notations related to the graphs we refer to [34], [101], [112] and [151].

Definition 1 : *A* graph *G consists of a finite non-empty set V(G) called the set of vertices*

together with a prescribed set $E(G)$ of unordered pairs of distinct vertices of G. The number of vertices and the number of edges of a graph G are denoted by p and q respectively. Each pair $[u,v] \in E(G)$ is said to be incident *with the vertices u and v and the vertices u and v are said to be* adjacent. *Two vertices u and v of G are* non-adjacent *if they are not adjacent. If two edges are incident with a common vertex then they are* adjacent edges. *Let v be a vertex of G and W be a subset of $V(G)$ not containing the vertex v. The set of all edges incident with v and a vertex in W is denoted by $[v,W]$. Let U and W be disjoint subsets of $V(G)$. The set of all edges incident with a vertex in U and a vertex in W is denoted by $[U,W]$.*

Definition 2 : *Let G be a graph. The* degree *of a vertex v in G is the number of edges incident with it and it is denoted by $deg_G(v)$. The maximum degree of G is denoted by $\Delta(G)$. If all the vertices of a graph have the same degree then it is called a* regular graph.

Definition 3 : *The* degree sequence *of a graph G is the sequence of the degrees of the p vertices of G arranged in non-increasing order which is denoted by $d_1 \geq d_2 \geq \cdots \geq d_p$.*

Definition 4 : *Let G be a graph. For a vertex v of G the set $Nhd_G(v)$ is the set of all vertices which are adjacent to v in G.*

Definition 5 : *A* subgraph *H of a graph G is a graph having all of its vertices and edges in G.*

Definition 6 : *Let G be a graph. For any set V' such that $V' \subseteq V(G)$, the* vertex induced subgraph *$< V' >$ is the maximal subgraph of G with V' as the vertex set.*

Definition 7 : *Let G be a graph. For any set E' such that $E' \subseteq E(G)$, the* edge induced subgraph *$< E' >$ is the subgraph of G with V' as its vertex set and E' as its edge set where V' consists of all the vertices incident with the edges in E'.*

Definition 8 : *A* walk *of a graph G is an alternating sequence of vertices and edges $v_0, e_1, v_1, e_2, v_2, \ldots, v_{n-1}, e_n, v_n$ beginning and ending with the vertices in which each edge is incident with the two vertices immediately preceding and following it. This walk* joins *v_0 and v_n. A walk is* closed *if $v_0 = v_n$. A walk is a* path *if all the vertices (and thus all the edges)*

are distinct. A path consisting of n vertices is denoted by P_n. A closed path is called a cycle. *A cycle consisting of n vertices is denoted by C_n. A vertex induced subgraph of a graph G which is a cycle is called an* induced cycle *of G.*

Definition 9 *: A graph is C_4-*free *if it does not have any induced C_4.*

Definition 10 *: A graph is* connected *if every pair of vertices is joined by a path. A maximal connected subgraph of a graph G is called a* component *of G.*

Definition 11 *: Let G be a graph. For a vertex v of G, the graph $G - v$ is obtained by deleting the vertex v and all the edges incident with it.*

Definition 12 *: Let G be a graph with 4n vertices. Let $U = \{v \in V(G) : deg_G(v) \geq 2n\}$. Let u_0 be a vertex not belonging to V(G). The graph $G_U \diamond u_0$ has $V(G) \cup \{u_0\}$ as its vertex set and $E(G) \cup E'$ as its edge set where E' is the set of edges obtained by joining u_0 to all the vertices of U.*

Consider the graph G with 8 vertices shown in Figure 1.1(a). The graph $G_U \diamond u_0$ where $U = \{v \in V(G) : deg_G(v) \geq 4\}$ and u_0 is a vertex not belonging to $V(G)$ is shown in Figure 1.1(b).

Definition 13 *: A bipartite graph G is a graph whose vertex set V(G) can be partitioned into two subsets V_1 and V_2 such that every edge of G is incident with a vertex of V_1 and a vertex of V_2.*

Definition 14 *: A graph is said to be* complete *if every pair of its vertices are adjacent. A complete graph with p vertices is denoted by K_p. The graph $2K_2$ has two components each of which is K_2. A* clique *of a graph is a maximal complete subgraph of the graph. A clique C of a graph G is maximum if there is no other clique of G with more number of vertices than the number of vertices in C. For a graph G the number of vertices in a maximum clique of G is called the* clique number *of G and it is denoted by $\omega(G)$.*

Definition 15 *: Let G be a graph. A subset S of G is a* stable set *(also called an independent set in the literature) if no two vertices of S are adjacent in G. A stable set of G which has*

maximum number of vertices in it is called a maximum stable set *of G. The number of vertices in a maximum stable set of G is called the* stability number *of G and it is denoted by $\alpha(G)$.*

Definition 16 *: A* clique cover *of order k of a graph G is a partition of its vertex set V(G) into k subsets $V_1, V_2, \ldots, V_k$ such that each $< V_i >$ is a clique of G. A clique cover of a graph G with smallest order is called a* minimum clique cover *of G. The order of a minimum clique cover of a graph G is called the* clique cover number *of G and it is denoted by $\theta(G)$.*

Definition 17 *: A* proper vertex coloring *of a graph G is assigning colors to the vertices of G such that no two adjacent vertices are assigned the same color. The* chromatic number *$\chi(G)$ of a graph G is the minimum number of colors needed to properly color the vertices of G.*

Definition 18 *: A* proper edge coloring *of a graph G is assigning colors to the edges of G such that no two adjacent edges are assigned the same color. The* chromatic index *$\chi'(G)$ of a graph G is the minimum number of colors needed to properly color the edges of G. A graph G is said to be a* Class 1 graph *if $\chi'(G) = \Delta(G)$ and a* Class 2 graph *if $\chi'(G) = \Delta(G) + 1$.*

Definition 19 *: A graph G is* χ-perfect *if $\chi(H) = \omega(H)$ for every induced subgraph H of G. A graph G is* α-perfect *if $\alpha(H) = \theta(H)$ for every induced subgraph H of G. A graph G is* perfect *if it is χ-perfect (or equivalently it is α-perfect).*

Definition 20 *: A graph is* chordal *if it has no induced cycle C_n for all $n \geq 4$.*

Definition 21 *: A graph is* split *if its vertex set can be partitioned into V' and S such that $< V' >$ is a clique of G and S is a stable set of G.*

Definition 22 *: Let G be a graph with p vertices whose vertices are labelled 1,2,...,p. For G its* adjacency matrix *is defined as the $p \times p$ matrix (a_{ij}) where a_{ij} is defined as follows.*

$$a_{ij} = \begin{cases} 1 & if\ [i,j] \in E(G) \\ 0 & otherwise \end{cases}$$

Definition 23 : *The* eigen values *of a graph G are the eigen values of the adjacency matrix of G obtained by labelling the vertices of G as 1,2,...,p. The eigenvalues of a graph constitute its* spectrum. *Two graphs are* cospectral *if they have the same spectrum. Maximum of the eigen values of a graph G is called the* maximum eigen value *of G and it is denoted by* $\lambda_{max}(G)$.

Definition 24 : *The* eigen vectors *of a graph G are the eigen vectors of the adjacency matrix of G obtained by labelling the vertices of G as 1,2,...,p.*

Definition 25 : *Let G be a graph. A* dominating set *of G is a subset D of V(G) such that each vertex of* $V(G) - D$ *is adjacent to atleast one vertex of D. A dominating set D of G is called a* minimum dominating set *if there is no other dominating set of G with lesser number of elements than D. The* domination number $\delta_0(G)$ *of G is the number of vertices in a minimum dominating set of G. A dominating set of G which is also a stable set is called a* kernel *of G. A kernel D of G is called a* minimum kernel *of G if there is no other kernel of G with lesser number of elements than D. The* independent domination number $\delta_i(G)$ *of G is the number of vertices in a minimum kernel of G.*

Definition 26 : *Two graphs* G' *and* G'' *are* isomorphic *if there exists a one-to-one correspondence between the vertex sets of* G' *and* G'' *which preserves adjacencies of the vertices. Such a one-to-one correspondence is called a* vertex isomorphism *of* G' *onto* G'' *and is denoted by* ψ. *If* G' *and* G'' *are isomorphic then we denote it by* $G' \cong G''$. *Two graphs are* non-isomorphic *if they are not isomorphic.*

Definition 27 : *Let* G' *and* G'' *be two isomorphic graphs and let* ψ *be an isomorphism of* G' *onto* G''. *For every subset U of* $V(G')$ *the set* $\psi(U) \subseteq V(G'')$ *is defined as* $\psi(U) = \{\psi(u) \in V(G'') : u \in U\}$.

Definition 28 : *The problem of verifying whether two given graphs are isomorphic or not is called the* isomorphism problem. *The isomorphism problem of a class of graphs* $\mathcal{F}$ *is called the isomorphism of* $\mathcal{F}$.

Definition 29 : *A* graph parameter *is a number or a set of numbers associated with graphs*

such that for any two isomorphic graphs the graph parameter is the same.

Definition 30 : *The* complement $\bar{G}$ *of a graph* G *has* $V(G)$ *as its vertex set and two vertices are adjacent in* $\bar{G}$ *if and only if they are non-adjacent in* G.

Definition 31 : *A graph* G *is* self-complementary (s.c.) *if it is isomorphic to its complement* $\bar{G}$. *The number of non-isomorphic s.c. graphs with* p *vertices is denoted by* s_p.

Definition 32 : *A* complementing permutation (c.p.) σ *of a s.c. graph* G *is a vertex isomorphism of* G *onto* $\bar{G}$. *A c.p.* σ *of* G *can be written as product of disjoint permutation cycles denoted by* $\sigma_1\sigma_2\cdots\sigma_s$ *since* G *and* $\bar{G}$ *have the same vertex set. Let* $J \subseteq \{1,2,\ldots,s\}$. *The product of the permutation cycles* $\sigma_i's$ *such that* $i \in J$ *is denoted by* $\Pi_{i\in J}\sigma_i$. *The product* $\Pi_{i=2}^{s}\sigma_i$ *is denoted by* σ/σ_1.

Definition 33 : *Let* G *be a s.c. graph and let* $\sigma = \sigma_1\sigma_2\cdots\sigma_s$ *where* $\sigma_i = (v_{i1}v_{i2}\cdots v_{ip_i})$ *be a c.p. of* G. *If* σ *has a single permutation cycle (that is when s=1) the vertices* $v_{11}, v_{12}, \ldots, v_{1p}$ *are identified with the labels 1,2,...,p respectively.*

Definition 34 : *Let* G *be a s.c. graph and* $\sigma = \sigma_1\sigma_2\cdots\sigma_s$ *where* $\sigma_i = (v_{i1}v_{i2}\ldots v_{ip_i})$ *be a c.p. of* G. *A vertex* v_{ij} *of* σ_i *is* even *or* odd *labelled if the subscript* j *of* v_{ij} *is even or odd respectively. The set consisting of all even labelled vertices of* σ_i *and the set consisting of all odd labelled vertices of* σ_i *are denoted by* $Even(\sigma_i)$ *and* $Odd(\sigma_i)$ *respectively. The sets* $Even(\sigma)$ *and* $Odd(\sigma)$ *are defined to be* $\cup_{i=1}^{s}Even(\sigma_i)$ *and* $\cup_{i=1}^{s}Odd(\sigma_i)$ *respectively. The sets* $Even(\sigma)$ *and* $Odd(\sigma)$ *are called the set of even labelled vertices of* σ *and the set of odd labelled vertices of* σ *respectively.*

Definition 35 : *Let* G *be a s.c. graph and let* $\sigma = \sigma_1\sigma_2\cdots\sigma_s$ *be a c.p. of* G. *The* length *of a permutation cycle* σ_i *is the number of vertices present in it.*

Definition 36 : *Let* G *be a s.c. graph and let* σ *be a c.p. of* G. *The vertex* $\sigma(v)$ *is the image of a vertex* v *under* σ. *The notation* σ^k *denotes* σ *multiplied* k *times. For a set* $U \subseteq V(G)$, $\sigma^k(U)$ *is defined as the set of all vertices* $\sigma^k(u)$ *such that* $u \in U$.

Let G be a s.c. graph with a c.p. σ. Ringel [178] and Sachs [190] proved that the

length of every permutation cycle of σ is divisible by 4 when p=4n and the length of every permutation cycle of σ is divisible by 4 except for a single permutation cycle of length 1 when p=4n+1.

Definition 37 : *Let G be a s.c. graph with p=4n. A c.p.* $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ *of G where* $\sigma_i^* = (v_{i1} v_{i2} \cdots v_{ip_i})$ *such that* $[v_{i2}, v_{i4}] \in E(G)$ *for all* $1 \leq i \leq s$ *is called a* star c.p. *of G.*

Consider the s.c. graph G shown in Figure 1.2. The permutation (2 3 4 1)(5 6 7 8) on the vertices of G is a c.p. whereas it is not a star c.p. since $[3,1] \notin E(G)$. The permutation (1 2 3 4) (5 6 7 8) is a star c.p. of G.

Definition 38 : *Let G be a s.c. graph with p=4n+1 and a c.p.* $\sigma = \sigma_1 \sigma_2 \cdots \sigma_s$. *We assume* σ_1 *to be the permutation cycle of length 1 and denote the vertex present in* σ_1 *by* v_0. *The vertex* v_0 *is called the fixed vertex of* σ.

Definition 39 : *Let G be a s.c. graph with p=4n+1. A c.p.* $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ *of G where* $\sigma_1^* = (v_0)$ *and* $\sigma_i^* = (v_{i1} v_{i2} \cdots v_{ip_i})$ *for all* $2 \leq i \leq s$ *is called a* star c.p. *of G if* $[v_{i2}, v_{i4}] \in E(G)$ *for all* $2 \leq i \leq s$.

Consider the s.c. graph G shown in Figure 1.3. The permutation (0)(2 3 4 1)(5 6 7 8) on the vertices of G is a c.p. whereas it is not a star c.p. since $[3,1] \notin E(G)$. The permutation (0)(1 2 3 4) (5 6 7 8) is a star c.p. of G.

Remark: In Section2.3 we prove that every s.c. graph has a star c.p. (Theorem 2.6).

Definition 40 : *A s.c. graph which is chordal is called a* s.c. chordal graph.

Definition 41 : *A s.c. graph which is perfect is called a* s.c. perfect graph.

Definition 42 : *A list of all non-isomorphic s.c. graphs with p vertices is called the* catalogue of s.c. graphs *with p vertices. A list of all non-isomorphic s.c. chordal graphs with p vertices is called the catalogue of s.c. chordal graphs with p vertices.*

Definition 43 : *An algorithm is said to run in O(f(x)) time if the time taken by the algorithm is atmost cf(x) where c is a positive integer and x is the size of the input. In this*

thesis for a s.c. chordal graph with 4n or 4n+1 vertices we always take n to be the size of the input.

Definition 44 : *An algorithm is said to be* polynomially computable *if the running time of the algorithm is $O(p(x))$ where $p(x)$ is a polynomial function. An algorithm is said to be computable in* linear time *if the running time of the algorithm is $O(p(x))$ where $p(x)$ is a polynomial of degree 1.*

Definition 45 : *The isomorphism of a class of graphs $\mathcal{F}_1$ is said to be* polynomially reducible *to the isomorphism of a class of graphs $\mathcal{F}_2$ if the following hold.*

(i) *There exists a polynomially computable algorithm which constructs a graph G' of $\mathcal{F}_2$ from a graph G of $\mathcal{F}_1$.*

(ii) *Two graphs G_1 and G_2 of $\mathcal{F}_1$ are isomorphic if and only if the graphs G_1' and G_2' of $\mathcal{F}_2$ (constructed by the polynomially computable algorithm) are isomorphic.*

Definition 46 : *The isomorphism of two classes of graphs $\mathcal{F}_1$ and $\mathcal{F}_2$ are polynomially equivalent if the isomorphism of $\mathcal{F}_1$ is polynomially reducible to the isomorphism of $\mathcal{F}_2$ and vice versa.*

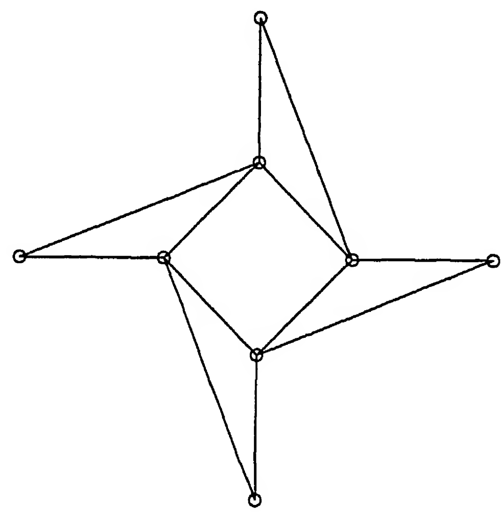

Figure 1.1a

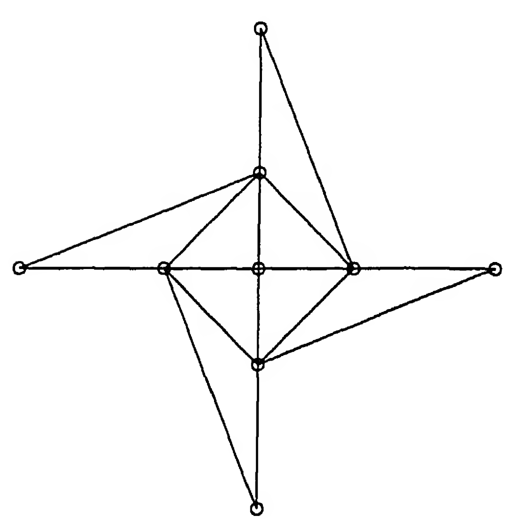

Figure 1.1b

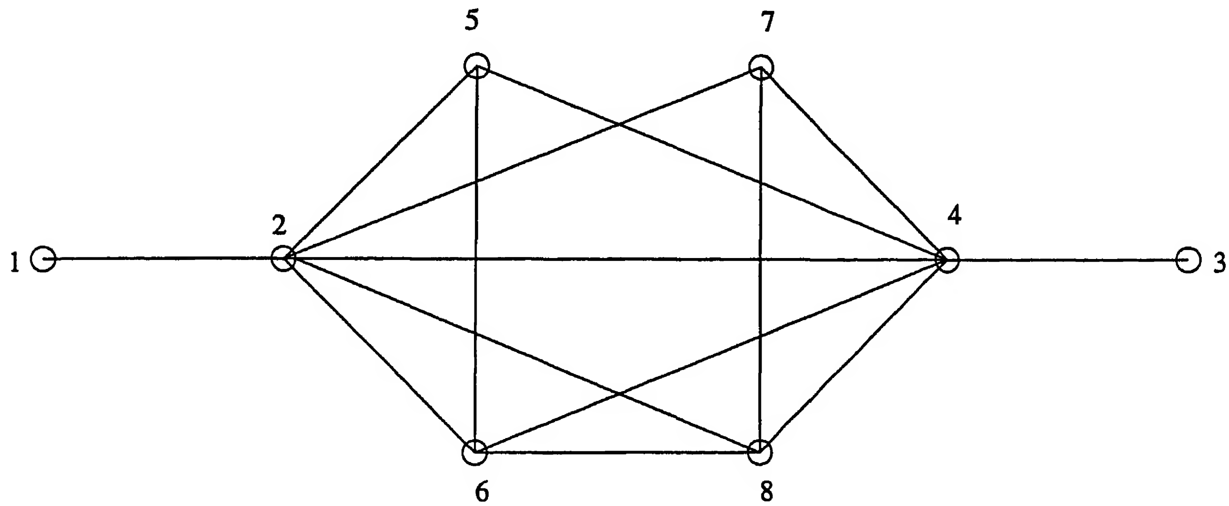

Figure 1.2

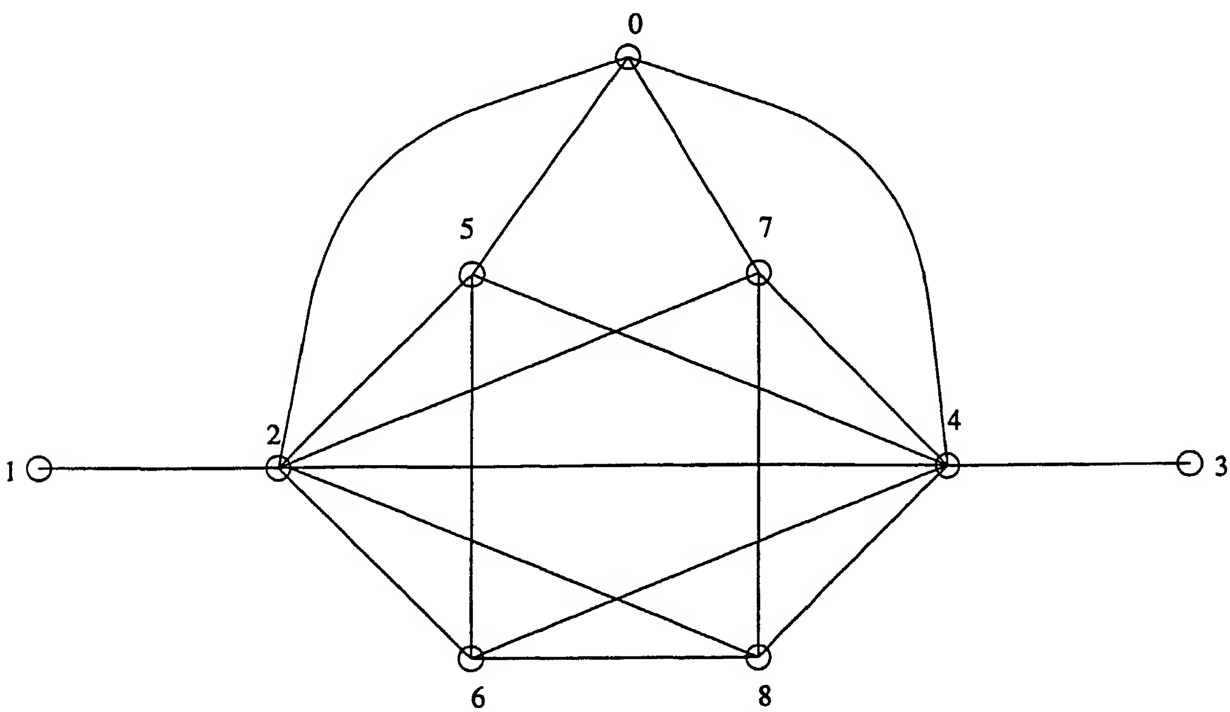

Figure 1.3

Chapter 2

On the characterisations for self-complementary graphs to be chordal

2.1 Introduction

In the literature various characterisations were obtained for chordal graphs and some classes of chordal graphs like split graphs, interval graphs etc. For these we refer to [10], [24], [39], [75], [85], [89], [95], [101], [109] and [229]. In this chapter we obtain the following characterisations.

(i) Let G be a s.c. graph. Then G is chordal if and only if $\omega(G) = 2n$ when $p = 4n$ and $\omega(G) = 2n+1$ when $p = 4n+1$ where p denotes the number of vertices of G and $\omega(G)$ denotes the clique number of G.

(ii) Let G be a s.c. graph. Then G is chordal if and only if $\alpha(G) = 2n$ when $p = 4n$ and $\alpha(G) = 2n+1$ when $p = 4n+1$ where p denotes the number of vertices of G and $\alpha(G)$ denotes the stability number of G.

(iii) Let G be a s.c. graph. Then G is chordal if and only if G has no induced subgraph isomorphic to C_4 when $p = 4n$ and G has no induced subgraph isomorphic to C_4 or

C_5 when $p = 4n+1$ where p denotes the number of vertices of G and C_n denotes the cycle with n vertices.

(iv) Let G be a s.c. graph with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_p$. Then G is chordal if and only if $\sum_{i=1}^{2n} d_i = 6n^2 - 2n$ when $p = 4n$ and $\sum_{i=1}^{2n} d_i = 6n^2$ when $p = 4n+1$ where p denotes the number of vertices of G.

The characterisations (i), (ii), (iii) and (iv) are obtained in Section 2.3, Section 2.4, Section 2.5 and Section 2.6 respectively by using the results stated in Section 2.2.

2.2 Basic Results

For obtaining the various characterisations for a self-complementary graph to be a chordal graph we use the following results available in the literature.

Ringel [178] and Sachs [190] independently obtained the following result on the structure of a c.p. of a s.c. graph.

Theorem 2.1 *(Ringel [178], Sachs [190]) : Let G be a s.c. graph with a c.p. σ. Then*

(i) *the order of every permutation cycle of σ is divisible by 4 if p=4n*

(ii) *the order of every permutation cycle of σ is divisible by 4, except for a single permutation cycle of order 1 if p=4n+1.*

Foldes and Hammer [85] had obtained the following characterisations for split graphs.

Theorem 2.2 *(Foldes and Hammer [85]) : Let G be a graph. Then G is a split graph if and only if G has no induced subgraph isomorphic to $2K_2$, C_4 or C_5.*

Theorem 2.3 *(Foldes and Hammer [85]) : Let G be a graph. Then G is a split graph if and only if both G and $\bar{G}$ are chordal graphs.*

The following results are due to Hammer and Simone [109].

Theorem 2.4 *(Hammer and Simone [109]) : Let G be a graph with the degree sequence* $d_1 \geq d_2 \geq \cdots \geq d_p$. *Let* $\mathcal{M} = max\{i : d_i \geq i-1,\ 1 \leq i \leq p\}$. *Then G is a split graph implies* $\omega(G) = \mathcal{M}$.

Theorem 2.5 *(Hammer and Simone [109]) : Let G be a graph with the degree sequence* $d_1 \geq d_2 \geq \cdots \geq d_p$. *Let* $\mathcal{M} = max\{i : d_i \geq i-1, 1 \leq i \leq p\}$. *Then G is a split graph if and only if* $\sum_{i=1}^{\mathcal{M}} d_i = \mathcal{M}(\mathcal{M}-1) + \sum_{i=\mathcal{M}+1}^{p} d_i$.

2.3 A characterisation for self-complementary graphs to be chordal in terms of the clique number

Lemma 2.1 : *Let G be a s.c. graph with a c.p.* σ. *Let u and v be two vertices of G. Then* $[u,v] \in E(G)$ *if and only if* $[\sigma(u), \sigma(v)] \notin E(G)$.

Proof: We note that $[u,v] \in E(G)$ if and only if $[\sigma(u), \sigma(v)] \notin E(\bar{G})$ for σ is an isomorphism of G onto $\bar{G}$. Also $[\sigma(u), \sigma(v)] \in E(\bar{G})$ if and only if $[\sigma(u), \sigma(v)] \notin E(G)$. Hence $[u,v] \in E(G)$ if and only if $[\sigma(u), \sigma(v)] \notin E(G)$. □

Lemma 2.2 : *Let G be a s.c. graph with a c.p.* σ. *Let u and v be two vertices of G. Then*

(i) $[u,v] \in E(G)$ *if and only if* $[\sigma^{2i}(u), \sigma^{2i}(v)] \in E(G)$

(ii) $[u,v] \in E(G)$ *if and only if* $[\sigma^{2i+1}(u), \sigma^{2i+1}(v)] \notin E(G)$

where i is a positive integer.

Proof: (i) Let u and v be two vertices of G. We prove that $[u,v] \in E(G)$ if and only if $[\sigma^{2i}(u), \sigma^{2i}(v)] \in E(G)$ for all positive integers i, by induction on i. By Lemma 2.1 $[u,v] \in E(G)$ if and only if $[\sigma(u), \sigma(v)] \notin E(G)$. Also by Lemma 2.1 $[\sigma(u), \sigma(v)] \notin E(G)$ if and only if $[\sigma^2(u), \sigma^2(v)] \in E(G)$. Hence $[u,v] \in E(G)$ if and only if $[\sigma^2(u), \sigma^2(v)] \in E(G)$. Therefore for $i = 1$ the result is true. Assume the result for $i = k$. That is $[u,v] \in E(G)$ if and only if $[\sigma^k(u), \sigma^k(v)] \in E(G)$. By Lemma 2.1 $[\sigma^{2k}(u), \sigma^{2k}(v)] \in E(G)$ if and only if $[\sigma^{2k+1}(u), \sigma^{2k+1}(v)] \notin E(G)$. Also by Lemma 2.1 $[\sigma^{2k+1}(u), \sigma^{2k+1}(v)] \notin E(G)$ if and

only if $[\sigma^{2k+2}(u), \sigma^{2k+2}(v)] \in E(G)$. Hence by the assumption $[u, v] \in E(G)$ if and only if $[\sigma^{2k+2}(u), \sigma^{2k+2}(v)] \in E(G)$. Therefore the result is true for $i = k + 1$.

(ii) By Lemma 2.1 $[u, v] \in E(G)$ if and only if $[\sigma(u), \sigma(v)] \notin E(G)$. By Lemma2.2(i) $[\sigma(u), \sigma(v)] \notin E(G)$ if and only if $[\sigma^{2i+1}(u), \sigma^{2i+1}(v)] \notin E(G)$ where i is a positive integer. Hence $[u, v] \in E(G)$ if and only if $[\sigma^{2i+1}(u), \sigma^{2i+1}(v)] \notin E(G)$ where i is a positive integer. □

Existence of a star c.p. for every s.c. graph follows from the following result.

Theorem 2.6 : *Let G be a s.c. graph. There exists a star c.p. σ^* of G.*

Proof: Let p=4n. Let $\sigma = \sigma_1\sigma_2 \cdots \sigma_s$ be a c.p. of G where $\sigma_i = (v_{i1}v_{i2} \cdots v_{ip_i})$ for all $1 \leq i \leq s$. By Lemma 2.1 either $[v_{i1}, v_{i3}] \in E(G)$ or $[\sigma(v_{i1}), \sigma(v_{i3})] = [v_{i2}, v_{i4}] \in E(G)$ for all $1 \leq i \leq s$. If $[v_{i2}, v_{i4}] \in E(G)$ for all $1 \leq i \leq s$, relabel the vertices $v_{i1}, v_{i2}, \ldots, v_{ip_i}$ as $v^*_{i1}, v^*_{i2}, \ldots, v^*_{ip_i}$ respectively, for all $1 \leq i \leq s$. Otherwise relabel the vertices $v_{i1}, v_{i2}, \ldots, v_{i(p_i-1)}, v_{ip_i}$ as $v^*_{i2}, v^*_{i3}, \ldots, v^*_{ip_i}, v^*_{i1}$ respectively, for all $1 \leq i \leq s$. Define $\sigma^* = \sigma^*_1\sigma^*_2 \cdots \sigma^*_s$ where $\sigma^*_i = (v^*_{i1}v^*_{i2} \cdots v^*_{ip_i})$ for all $1 \leq i \leq s$. We note that $[v^*_{i2}, v^*_{i4}] \in E(G)$ for all $1 \leq i \leq s$ since $[v^*_{i2}, v^*_{i4}] = [v_{i2}, v_{i4}]$ if $[v_{i2}, v_{i4}] \in E(G)$ and $[v^*_{i2}, v^*_{i4}] = [v_{i1}, v_{i3}]$ if $[v_{i1}, v_{i3}] \in E(G)$ for all $1 \leq i \leq s$. Also σ^* is a c.p. of G since σ is a c.p. of G.

Let p=4n+1. Let $\sigma = \sigma_1\sigma_2 \cdots \sigma_s$ be a c.p. of G where $\sigma_1 = (v_0)$ and $\sigma_i = (v_{i1}v_{i2} \cdots v_{ip_i})$ for all $2 \leq i \leq s$. By Lemma 2.1 either $[v_{i1}, v_{i3}] \in E(G)$ or $[\sigma(v_{i1}), \sigma(v_{i3})] = [v_{i2}, v_{i4}] \in E(G)$ for all $2 \leq i \leq s$. If $[v_{i2}, v_{i4}] \in E(G)$ for all $2 \leq i \leq s$, relabel the vertices $v_{i1}, v_{i2}, \ldots, v_{ip_i}$ as $v^*_{i1}, v^*_{i2}, \ldots, v^*_{ip_i}$ respectively, for all $2 \leq i \leq s$. Otherwise relabel the vertices $v_{i1}, v_{i2}, \ldots, v_{i(p_i-1)}, v_{ip_i}$ as $v^*_{i2}, v^*_{i3}, \ldots, v^*_{ip_i}, v^*_{i1}$ respectively, for all $2 \leq i \leq s$. Define $\sigma^* = \sigma^*_1\sigma^*_2 \cdots \sigma^*_s$ where $\sigma^*_i = (v^*_{i1}v^*_{i2} \cdots v^*_{ip_i})$ for all $2 \leq i \leq s$. We note that $[v^*_{i2}, v^*_{i4}] \in E(G)$ for all $2 \leq i \leq s$ since $[v^*_{i2}, v^*_{i4}] = [v_{i2}, v_{i4}]$ if $[v_{i2}, v_{i4}] \in E(G)$ and $[v^*_{i2}, v^*_{i4}] = [v_{i1}, v_{i3}]$ if $[v_{i1}, v_{i3}] \in E(G)$ for all $2 \leq i \leq s$. Also σ^* is a c.p. of G since σ is a c.p. of G. □

The following result determines the degree sequence of the complement of a graph G from the the degree sequence of G.

Theorem 2.7 : *Let G be a graph with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_p$. Then the degree sequence of $\bar{G}$ is $p - 1 - d_p \geq p - 1 - d_{p-1} \geq \cdots \geq p - 1 - d_1$.*

Proof: Let v be a vertex of G with degree d. Then p-1-d is the degree of v in $\bar{G}$. $\square$

The following result establishes a relation between the ith term and the $(p-i+1)$th term of the degree sequence of a s.c. graph.

Theorem 2.8 : *Let G be a s.c. graph with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_p$. Then $d_i + d_{p-i+1} = p-1$ where $1 \leq i \leq p$.*

Proof: By Theorem 2.7 $p-1-d_p \geq p-1-d_{p-1} \geq \cdots \geq p-1-d_1$ is the degree sequence of G since G is a s.c. graph. Hence $d_i = p-1-d_{p-i+1}$ for all $1 \leq i \leq p$. $\square$

For a s.c. graph G with the degree sequence $d_1 \geq d_2 \geq \cdots \geq d_p$ the following result determines the value of $\mathcal{M}$ where $\mathcal{M} = max\{i : d_i \geq i-1,\ 1 \leq i \leq p\}$.

Theorem 2.9 : *Let G be a s.c. graph with the degree sequence*
$d_1 \geq d_2 \geq \cdots \geq d_p$. Let $\mathcal{M} = max\{i : d_i \geq i-1,\ 1 \leq i \leq p\}$. Then

(i) *$\mathcal{M} = 2n$ if p=4n*

(ii) *$\mathcal{M} = 2n+1$ if p=4n+1.*

Proof: (i) Let p=4n. We prove that $d_i \geq 2n$ for all $1 \leq i \leq 2n$ and $d_i \leq 2n-1$ for all $2n+1 \leq i \leq 4n$. Let $d_k \leq 2n-1$ for some $1 \leq k \leq 2n$. By Theorem 2.8 $d_{p-k+1} \geq 2n$ since $d_k \leq 2n-1$. Also $d_k \geq d_{p-k+1}$ since $1 \leq k \leq 2n$. Hence $2n-1 \geq 2n$, a contradiction. Therefore $d_i \geq 2n$ for all $1 \leq i \leq 2n$. Let $d_k \geq 2n-1$ for some $2n+1 \leq k \leq 4n$. By Theorem2.8 $d_{p-k+1} \leq 2n-1$ since $d_k \geq 2n$. Also $d_{p-k+1} \geq d_k$ since $2n+1 \leq k \leq 4n$. Hence $2n-1 \geq 2n$, a contradiction. Therefore $d_i \leq 2n-1$ for all $2n+1 \leq i \leq 4n$. We note that $\mathcal{M} = max\{i : d_i \geq i-1, 1 \leq i \leq p\} = 2n$ since $d_i \geq 2n$ for all $1 \leq i \leq 2n$ and $d_i \leq 2n-1$ for all $2n+1 \leq i \leq 4n$.

(ii) Let p=4n+1. We prove that $d_i \geq 2n$ for all $1 \leq i \leq 2n+1$ and $d_i \leq 2n$ for all $2n+1 \leq i \leq 4n+1$. Let $d_k \leq 2n-1$ for some $1 \leq k \leq 2n+1$. By Theorem 2.8 $d_{p-k+1} \geq 2n+1$ since $d_k \leq 2n-1$. Also $d_{p-k+1} \leq d_k$ since $1 \leq k \leq 2n+1$. Hence $2n-1 \geq 2n+1$, a contradiction. Therefore $d_i \geq 2n$ for all $1 \leq i \leq 2n+1$. Let $d_k \geq 2n+1$ for some $2n+1 \leq k \leq 4n+1$. By Theorem 2.8 $d_{p-k+1} \leq 2n-1$ since $d_k \geq 2n+1$. Also $d_k \leq d_{p-k+1}$ since $2n+1 \leq k \leq 4n+1$. Hence $2n-1 \geq 2n+1$, a contradiction. Therefore $d_i \leq 2n$ for

all $2n+1 \leq i \leq 4n+1$. We note that $\mathcal{M} = max\{i : d_i \geq i-1, 1 \leq i \leq p\} = 2n+1$.□

The following result gives a necessary condition for a s.c. graph to be a split graph in terms of its clique number.

Theorem 2.10 : *Let G be a s.c graph. Then G is a split graph implies*
(i) $\omega(G) = 2n$ *if p= 4n*
(ii) $\omega(G) = 2n+1$ *if p=4n+1.*

Proof: Follows from Theorem 2.4 and Theorem 2.9. □

Lemma 2.3 : *Let G be a s.c. graph. Let H be an induced subgraph of G. Then there exists an induced subgraph of G isomorphic to* $\bar{H}$.

Proof: Let σ be a c.p. of G. Let V_H be the vertex set of H. We note that $< \sigma(V_H) >$ is an induced subgraph of G isomorphic to $\bar{H}$ since σ is a c.p. . □

An upper bound for the clique number of a s.c. graph is obtained in the following Theorem.

Theorem 2.11 : *Let G be a s.c. graph. Then*

(i) $\omega(G) \leq 2n$ *if p=4n*

(ii) $\omega(G) \leq 2n+1$ *if p=4n+1.*

Proof: (i) Let p=4n. Let $\omega(G) \geq 2n+1$. Let C be a maximum clique of G. Let K be a complete subgraph of C with 2n+1 vertices. By Lemma2.3 there exists a subgraph S of G isomorphic to $\bar{K}$. Let V_K be the vertex set of K. We note that V_K and S can have atmost one common vertex since K is a complete subgraph of G and S is a stable set of G. Hence $|V_K \cup S| \geq 4n+1$, thus a contradiction since $|V_K \cup S| \leq 4n$. Therefore $\omega(G) \leq 2n$.
(ii) Let p=4n+1. Let $\omega(G) \geq 2n+2$. Let C be a maximum clique of G. Let K be a complete subgraph of C with 2n+2 vertices. By Lemma2.3 there exists a subgraph S of G isomorphic to $\bar{K}$. Let V_K be the vertex set of K. We note that V_K and S can have atmost one common vertex since K is a complete subgraph of G and S is a stable set of G. Hence $|V_K \cup S| \geq 4n+3$, thus a contradiction since $|V_K \cup S| \leq 4n+1$. Therefore $\omega(G) \leq 2n+1$.□

Theorem 2.12 : *Let G be a s.c. graph with a c.p.* $\sigma = \sigma_1\sigma_2\cdots\sigma_s$ *where* $\sigma_i = (v_{i1}v_{i2}\cdots v_{ip_i})$ *for all* $1 \leq i \leq s$. *Let* $X_i = \{v_{ij} \in \sigma_i\}$ *where* $1 \leq i \leq s$ *and let* $Y_0 = \{1, 2, \ldots, s\}$. *Then* $< \bigcup_{i \in J} X_i >$ *is a s.c. graph with a c.p.* $\sigma_J = \Pi_{i \in J}\sigma_i$ *where* $J \subseteq Y_0$.

Proof: Choose $J \subseteq Y_0 = \{1, 2, \ldots, s\}$. We note that $\sigma_J = \Pi_{i \in J}\sigma_i$ is a c.p. of $< \bigcup_{i \in J} X_i >$ since σ is a c.p. of G. Hence $< \bigcup_{i \in J} X_i >$ is a s.c. graph. □

Corollary 2.1 : *Let G be a s.c. graph with p=4n+1. Let* $\sigma = \sigma_1\sigma_2\cdots\sigma_s$ *be a c.p. of G with* v_0 *as its fixed vertex. Then* $G - v_0$ *is a s.c. graph with a c.p.* σ/σ_1.

Proof: Follows from Theorem2.12. □

Theorem 2.13 : *Let G be a s.c. graph with a star c.p.* $\sigma^* = \sigma_1^*\sigma_2^*\cdots\sigma_s^*$ *where* $\sigma_i^* = (v_{i1}v_{i2}\cdots v_{ip_i})$ *for all* $1 \leq i \leq s$. *Let* $X_i = \{v_{ij} \in \sigma_i^*\}$ *where* $1 \leq i \leq s$ *and let* $Y_0 = \{1, 2, \ldots, s\}$. *Then* $< \bigcup_{i \in J} X_i >$ *is a s.c. graph with a star c.p.* $\sigma_J^* = \Pi_{i \in J}\sigma_i^*$ *where* $J \subseteq Y_0$.

Proof: Follows from the definition of star c.p. of a s.c. graph and Theorem 2.12. □

Corollary 2.2 : *Let G be a s.c. graph with p=4n+1. Let* $\sigma^* = \sigma_1^*\sigma_2^*\cdots\sigma_s^*$ *be a star c.p. of G with* v_0 *as its fixed vertex. Then* $G - v_0$ *is a s.c. graph with a star c.p.* σ^*/σ_1^*.

Proof: Follows from Theorem2.13. □

Lemma 2.4 : *Let G be a s.c. graph with a c.p.* $\sigma = \sigma_1\sigma_2\cdots\sigma_s$ *where* $\sigma_i = (v_{i1}v_{i2}\cdots v_{ip_i})$ *for all* $1 \leq i \leq s$. *Let* V' *be a subset of V such that* $< V' >$ *is a complete subgraph of G. If* $v_{im} \in V'$ *and* $\sigma(v_{im}) \in V'$ *for some vertex* v_{im} *of G then either* $v_{jn} \notin V'$ *or* $\sigma(v_{jn}) \notin V'$ *for every vertex* v_{jn} *of G distinct from* v_{im} *where* $1 \leq i \leq s$, $1 \leq j \leq s$, $1 \leq m \leq p_i$ *and* $1 \leq n \leq p_j$.

Proof: Let v_{im}, $\sigma(v_{im})$, v_{jn} and $\sigma(v_{jn})$ belong to V' where v_{im} and v_{jn} are two distinct vertices of G such that and $1 \leq i \leq s$, $1 \leq j \leq s$, $1 \leq m \leq p_i$ and $1 \leq n \leq p_j$. Then $[v_{im}, v_{jn}] \in E(G)$ and $[\sigma(v_{im}), \sigma(v_{jn})] \in E(G)$ since $< V' >$ is a complete subgraph of G. By Lemma2.1 $[\sigma(v_{im}), \sigma(v_{jn})] \notin E(G)$ since $[v_{im}, v_{jn}] \in E(G)$, a contradiction. □

Lemma 2.5 : *Let G be a s.c. graph with p=4n,* $\omega(G) = 2n$ *and a c.p.* $\sigma = (1\ 2\cdots 4n)$

where $n \geq 2$. Let C be a maximum clique of G and V' be the vertex set of C. Then there exists no vertex i of G such that $i \in V'$ and $\sigma(i) \in V'$ where $1 \leq i \leq 4n$.

Proof: Let $k \in V'$ and $\sigma(k) \in V'$ for some vertex $1 \leq k \leq 4n$. We note that $[k, \sigma(k)] \in E(G)$ since $k \in V'$ and $\sigma(k) \in V'$. By Lemma2.1 $[\sigma(k), \sigma^2(k)] \notin E(G)$ and by Lemma2.2 $[\sigma^{4n-1}(k), \sigma^{4n}(k)] = [\sigma^{4n-1}(k), k] \notin E(G)$ since $[k, \sigma(k)] \in E(G)$. Hence $\sigma^2(k) \notin V'$ and $\sigma^{4n-1}(k) \notin V'$. By Lemma2.2 either $[\sigma(k), \sigma^3(k)] \notin E(G)$ or $[\sigma^{4n-2}(k), \sigma^{4n}(k)] = [\sigma^{4n-2}(k), k] \notin E(G)$. Hence either $\sigma^3(k) \notin V'$ or $\sigma^{4n-2}(k) \notin V'$ since $k \in V'$ and $\sigma(k) \in V'$. By Lemma 2.4 atmost one vertex of each set $\{\sigma^4(k), \sigma^5(k)\}, \{\sigma^6(k), \sigma^7(k)\}, \ldots, \{\sigma^{4n-4}(k), \sigma^{4n-3}(k)\}$ belongs to V' since $k \in V'$ and $\sigma(k) \in V'$. Hence either $\sigma^3(k)$ or $\sigma^{4n-2}(k)$ but not both and exactly one vertex of each set $\{\sigma^4(k), \sigma^5(k)\}, \{\sigma^6(k), \sigma^7(k)\}, \ldots, \{\sigma^{4n-4}(k), \sigma^{4n-3}(k)\}$ belongs to V' since $|V'| = 2n$. Let $\sigma^3(k) \in V'$. By Lemma2.2 $[\sigma^3(k), \sigma^4(k)] \notin E(G), [\sigma^5(k), \sigma^6(k)] \notin E(G), \ldots, [\sigma^{4n-5}(k), \sigma^{4n-4}(k)] \notin E(G)$ since $[k, \sigma(k)] \in E(G)$. Hence $\sigma^3(k) \Rightarrow \sigma^5(k) \Rightarrow \sigma^7(k) \Rightarrow \cdots \Rightarrow \sigma^{4n-3}(k) \in V'$. Therefore $\sigma^3(k) \in V'$ and $\sigma^{4n-3}(k) \in V'$. By Lemma2.2 $[k, \sigma^3(k)] \notin E(G)$ or $[\sigma^{4n-3}(k), \sigma^{4n}(k)] = [\sigma^{4n-3}(k), k] \notin E(G)$. Hence $\sigma^3(k) \notin V'$ or $\sigma^{4n-3}(k) \notin V'$ since $k \in V'$, thus a contradiction to the fact that both $\sigma^3(k) \in V'$ and $\sigma^{4n-3}(k) \in V'$. Therefore $\sigma^3(k) \notin V'$. Then $\sigma^{4n-2}(k) \in V'$ since exactly one vertex of the vertices $\sigma^3(k)$ and $\sigma^{4n-2}(k)$ belong to V'. By Lemma2.2 $[\sigma^5(k), \sigma^6(k)] \notin E(G), [\sigma^7(k), \sigma^8(k)] \notin E(G), \ldots, [\sigma^{4n-3}(k), \sigma^{4n-2}(k)] \notin E(G)$ since $[k, \sigma(k)] \notin E(G)$. Hence $\sigma^{4n-2}(k) \Rightarrow \sigma^{4n-4}(k) \Rightarrow \cdots \Rightarrow \sigma^4(k) \in V'$. Therefore $\sigma^4(k) \in V'$ and $\sigma^{4n-2}(k) \in V'$. By Lemma 2.2 $[\sigma(k), \sigma^4(k)] \notin E(G)$ or $[\sigma^{4n-2}(k), \sigma^{4n+1}(k)] = [\sigma^{4n-2}(k), \sigma(k)] \notin E(G)$. Hence either $\sigma^4(k) \notin V'$ or $\sigma^{4n-2}(k) \notin V'$ since $\sigma(k) \in V'$, thus a contradiction to the fact that both $\sigma^4(k)$ and $\sigma^{4n-2}(k)$ belong to V'. Therefore there exists no vertex i of G such that $i \in V'$ and $\sigma(i) \in V'$ for all $1 \leq i \leq 4n$. □

Lemma 2.6 : *Let G be a s.c. graph with p=4n, $\omega(G) = 2n$ and a c.p. $\sigma = (1\ 2 \cdots 4n)$ where $n \geq 2$. Let C be a maximum clique of G and V' be the vertex set of C. Then either $V' = Even(\sigma)$ or $V' = Odd(\sigma)$.*

Proof: By Lemma 2.1 either $[1, 3] \notin E(G)$ or $[\sigma(1), \sigma(3)] = [2, 4] \notin E(G)$.

Case i : We prove that $V' = Even(\sigma)$ when $[1, 3] \notin E(G)$. Let $[1, 3] \notin E(G)$. We note that

$|V'| = 2n$ since $\omega(G) = 2n$. Also $|Even(\sigma)| = 2n$ and $Even(\sigma) \cap Odd(\sigma) = \emptyset$. Hence to prove $V' = Even(\sigma)$ we prove that V' does not have any odd labelled vertex. Let an odd labelled vertex $2r+1 \in V'$ for some $0 \leq r \leq 2n-1$. By Lemma2.2 either $[2r+1, \sigma(2r+1)] \in E(G)$ or $[\sigma^{4n-1}(2r+1), \sigma^{4n}(2r+1)] = [\sigma^{4n-1}(2r+1), 2r+1] \in E(G)$. Let $[2r+1, \sigma(2r+1)] \in E(G)$. By Lemma2.5 exactly one vertex of each set $\{2r+1, \sigma(2r+1)\}, \{\sigma^2(2r+1), \sigma^3(2r+1)\}, \ldots,$ $\{\sigma^{4n-2}(2r+1), \sigma^{4n-1}(2r+1)\}$ belong to V' since $|V'| = 2n$ and $\sigma^{4n}(2r+1) = 2r+1$. By Lemma2.2 $[\sigma^3(2r+1), \sigma^4(2r+1)] \notin E(G), [\sigma^5(2r+1), \sigma^6(2r+1)] \notin E(G), \ldots,$ $[\sigma^{4n-1}(2r+1), \sigma^{4n}(2r+1)] = [\sigma^{4n-1}(2r+1), 2r+1] \notin E(G)$, since $[2r+1, \sigma(2r+1)] \in E(G)$. Hence $2r+1 \Rightarrow \sigma^{4n-2}(2r+1) \Rightarrow \sigma^{4n-4}(2r+1) \Rightarrow \cdots \Rightarrow \sigma^2(2r+1) \in V'$. Therefore the 2n vertices of C are $2r+1, \sigma^2(2r+1), \ldots, \sigma^{4n-2}(2r+1)$ since the vertices $2r+1, \sigma^2(2r+1), \ldots, \sigma^{4n-2}(2r+1)$ are mutually distinct. Also $\sigma^{2i}(2r+1)$ is odd labelled where $1 \leq i \leq 2n-1$. Hence $V' = Odd(\sigma)$ since $|Odd(\sigma)| = 2n$, thus a contradiction to the fact that $[1,3] \notin E(G)$. Therefore $[2r+1, \sigma(2r+1)] \notin E(G)$. Then $[\sigma^{4n-1}(2r+1), 2r+1] \in E(G)$ since either $[2r+1, \sigma(2r+1)] \in E(G)$ or $[\sigma^{4n-1}(2r+1), 2r+1] \in E(G)$. By Lemma 2.5 exactly one vertex of each set $\{\sigma(2r+1), \sigma^2(2r+1)\}, \{\sigma^3(2r+1), \sigma^4(2r+1)\}, \ldots,$ $\{\sigma^{4n-1}(2r+1), \sigma^{4n}(2r+1)\} = \{\sigma^{4n-1}(2r+1), 2r+1\}$ belong to V' since $|V'| = 2n$. By Lemma2.2 $[2r+1, \sigma(2r+1)] \notin E(G), [\sigma^2(2r+1), \sigma^3(2r+1)] \notin E(G), \ldots,$ $[\sigma^{4n-2}(2r+1), \sigma^{4n-1}(2r+1)] \notin E(G)$ since $[\sigma^{4n-1}(2r+1), 2r+1] \in E(G)$ and $\sigma^{4n}(2r+1) = 2r+1$. Hence $2r+1 \Rightarrow \sigma^2(2r+1) \Rightarrow \sigma^4(2r+1) \Rightarrow \cdots \Rightarrow \sigma^{4n-2}(2r+1) \in V'$. Therefore the 2n vertices of C are $2r+1, \sigma^2(2r+1), \ldots, \sigma^{4n-2}(2r+1)$ since the vertices $2r+1, \sigma^2(2r+1), \ldots, \sigma^{4n-2}(2r+1)$ are mutually distinct. Also $\sigma^{2i}(2r+1)$ is odd labelled where $1 \leq i \leq 2n-1$. Hence $V' = Odd(\sigma)$ since $|Odd(\sigma)| = 2n$, thus a contradiction to the fact that $[1,3] \notin E(G)$. Therefore V' does not have any odd labelled vertex. Hence $V' = Even(\sigma)$ when $[1,3] \notin E(G)$.

Case ii : We prove that $V' = Odd(\sigma)$ when $[2,4] \notin E(G)$. Let $[2,4] \notin E(G)$. We note that $|V'| = 2n$ since $\omega(G) = 2n$. Also $|Odd(\sigma)| = 2n$ and $Odd(\sigma) \cap Even(\sigma) = \emptyset$. Hence to show $V' = Odd(\sigma)$ we prove that V' does not have any even labelled vertex. Let an even labelled vertex $2r \in V'$ for some $1 \leq r \leq 2n$. By Lemma2.2 either $[2r, \sigma(2r)] \in E(G)$ or $[\sigma^{4n-1}(2r), \sigma^{4n}(2r)] = [\sigma^{4n-1}(2r), 2r] \in E(G)$. Let $[2r, \sigma(2r)] \in E(G)$. By Lemma 2.5 exactly one vertex of each set $\{2r, \sigma(2r)\}, \{\sigma^2(2r), \sigma^3(2r)\}, \ldots, \{\sigma^{4n-2}(2r), \sigma^{4n-1}(2r)\}$ belongs to V'

since $|V'| = 2n$ and $\sigma^{4n}(2r) = 2r$. By Lemma2.2 $[\sigma^3(2r), \sigma^4(2r)] \notin E(G), [\sigma^5(2r), \sigma^6(2r)] \notin E(G), \ldots, [\sigma^{4n-1}(2r), \sigma^{4n}(2r)] = [\sigma^{4n-1}(2r), 2r] \notin E(G)$ since $[2r, \sigma(2r)] \in E(G)$. Hence $2r \Rightarrow \sigma^{4n-2}(2r) \Rightarrow \sigma^{4n-4}(2r) \Rightarrow \cdots \Rightarrow \sigma^2(2r) \in V'$. Therefore the 2n vertices of C are $2r, \sigma^2(2r), \ldots, \sigma^{4n-2}(2r)$ since the vertices $2r, \sigma^2(2r), \ldots, \sigma^{4n-2}(2r)$ are mutually distinct. Also $\sigma^{2i}(2r)$ is even labelled where $1 \leq i \leq 2n-1$. Hence $V' = Even(\sigma)$ since $|Even(\sigma)| = 2n$, thus a contradiction to the fact that $[2,4] \notin E(G)$. Therefore $[2r, \sigma(2r)] \notin E(G)$. Then $[\sigma^{4n-1}(2r), 2r] \in E(G)$ since either $[2r, \sigma(2r)] \in E(G)$ or $[\sigma^{4n-1}(2r), 2r] \in E(G)$. By Lemma 2.5 exactly one vertex of each set $\{\sigma(2r), \sigma^2(2r)\}, \{\sigma^3(2r), \sigma^4(2r)\}, \ldots, \{\sigma^{4n-1}(2r), \sigma^{4n}(2r)\} = \{\sigma^{4n-1}(2r), 2r\}$ belongs to V' since $|V'| = 2n$. By Lemma2.2 $[2r, \sigma(2r)] \notin E(G), [\sigma^2(2r), \sigma^3(2r)] \notin E(G), \ldots, [\sigma^{4n-2}(2r), \sigma^{4n-1}(2r)] \notin E(G)$ since $[\sigma^{4n-1}(2r), 2r] \in E(G)$ and $\sigma^{4n}(2r) = 2r$. Hence $2r \Rightarrow \sigma^2(2r) \Rightarrow \sigma^4(2r) \Rightarrow \cdots \Rightarrow \sigma^{4n-2}(2r) \in V'$. Therefore the 2n vertices of C are $2r, \sigma^2(2r), \ldots, \sigma^{4n-2}(2r)$ since the vertices $2r, \sigma^2(2r), \ldots, \sigma^{4n-2}(2r)$ are mutually distinct. Also $\sigma^{2i}(2r)$ is even labelled where $1 \leq i \leq 2n-1$. Hence $V' = Even(\sigma)$, since $|Even(\sigma)| = 2n$, thus a contradiction to the fact that $[2,4] \notin E(G)$. Therefore V' does not have any even labelled vertex. Hence $V' = Odd(\sigma)$ when $[2,4] \notin E(G)$. □

Corollary 2.3 : *Let G be a s.c. graph with p=4n, $\omega(G) = 2n$ and a star c.p. $\sigma^* = (1\ 2 \cdots 4n)$ where $n \geq 2$. Then $< Even(\sigma^*) >$ is the unique maximum clique of G.*

Proof: Let C be a maximum clique of G with V' as its vertex set. By Lemma2.6 $V' = Even(\sigma^*)$ or $V' = Odd(\sigma^*)$. We note that $[2,4] \in E$ since σ^* is a star c.p. . Hence by Lemma2.1 $[\sigma^*(2), \sigma^*(4)] = [3,5] \notin E$. Therefore $V' \neq Odd(\sigma^*)$. □

For a s.c. graph G with 4n vertices and a star c.p. σ^* the collection of all even labelled vertices of σ^* induces a maximum clique of G when $\omega(G) = 2n$.

Theorem 2.14 : *Let G be a s.c. graph with p=4n, $\omega(G) = 2n$ and a star c.p. σ^*. Then $< Even(\sigma^*) >$ is a maximum clique of G.*

Proof: Let p=4. In this case $G \cong P_4$. By Theorem 2.1 $\sigma^* = (1\ 2 \cdot 3\ 4)$ is the only star c.p. of G. Also $< Even(\sigma^*) >$ is a maximum clique of G since $[2,4] \in E(G)$. Hence the Theorem is true when p=4. Let $p \geq 8$. Let $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ where $\sigma_i^* = (v_{i1} v_{i2} \cdots v_{ip_i})$ for all $1 \leq i \leq s$. By Theorem 2.1 each p_i is divisible by 4. Let $V_i = \{v_{ij} : 1 \leq j \leq p_i\}$ where

$1 \leq i \leq s$. We note that $|V_i| = p_i$ where $1 \leq i \leq s$. Let C be a maximum clique of G and let V' be its vertex set. We prove that $|V' \cap V_i| = \frac{p_i}{2}$ for all $1 \leq i \leq s$. Let $|V' \cap V_j| \neq \frac{p_j}{2}$ for some $1 \leq j \leq s$. Then there exists a V_k such that $|V' \cap V_k| \geq \frac{p_k}{2} + 1$ for some $1 \leq k \leq s$. By Theorem2.13 $< V_k >$ is a s.c. graph. Hence by Theorem 2.11 every complete subgraph of $< V_k >$ has atmost $\frac{p_k}{2}$ vertices, thus a contradiction since $< V' \cap V_k >$ is a complete subgraph of $< V_k >$. Therefore $|V' \cap V_i| = \frac{p_i}{2}$ for all $1 \leq i \leq s$. By Corollary2.3 C has all even labelled vertices of $\sigma_i^{*'} s$ whose length is atleast 8 since $< V_i \cap V' >$ is a maximum clique of $< V_i >$ where $1 \leq i \leq s$. We prove that C has both an odd labelled vertex and an even labelled vertex from atmost one permutation cycle of length 4. Let C have both an even labelled and an odd labelled vertex from two distinct permutation cycles σ_m^* and σ_n^* each of length 4. Either $\sigma^*(v_{i2x}) = v_{i(2y+1)}$ or $\sigma^*(v_{i(2y+1)}) = v_{i2x}$ for any even labelled vertex v_{i2x} and for any odd labelled vertex $v_{i(2y+1)}$ of σ_i^* where σ_i^* is a permutation cycle of length 4 and $1 \leq i \leq s$. Therefore either of the cases (a), (b), (c) or (d) holds.

(a) There exist even labelled vertices v_{l2x} and v_{m2y} of σ_l^* and σ_m^* respectively such that v_{l2x}, $\sigma^*(v_{l2x})$, v_{m2y} and $\sigma^*(v_{m2y})$ belong to V'.

(b) There exist odd labelled vertices $v_{l(2x+1)}$ and $v_{m(2y+1)}$ of σ_l^* and σ_m^* respectively such that $v_{l(2x+1)}$, $\sigma^*(v_{l(2x+1)})$, $v_{m(2y+1)}$ and $\sigma^*(v_{m(2y+1)})$ belong to V'.

(c) There exist an even labelled vertex v_{l2x} of σ_l^* and an odd labelled vertex $v_{m(2y+1)}$ of σ_m^* such that v_{l2x}, $\sigma^*(v_{l2x})$, $v_{m(2y+1)}$ and $\sigma^*(v_{m(2y+1)})$ belong to V'.

(d) There exist an odd labelled vertex $v_{l(2x+1)}$ of σ_l^* and an even labelled vertex v_{m2y} of σ_m^* such that $v_{l(2x+1)}$, $\sigma^*(v_{l(2x+1)})$, v_{m2y} and $\sigma^*(v_{m2y})$ belong to V'.

All the cases (a), (b), (c) and (d) contradict Lemma2.4. So C has both an even labelled vertex and an odd labelled vertex from atmost one permutation cycle of length 4. We note that C cannot have the two odd labelled vertices of any permutation cycle of length 4 since σ^* is a star c.p. Hence C has only even labelled vertices of σ^* with a possible exception of a single odd labelled vertex of a permutation cycle σ_m^* of length 4 belonging to C. If C has only even labelled vertices of σ^* the Theorem is proved since C has 2n vertices. Otherwise an even labelled vertex v_{m2x} and an odd labelled vertex v_{m2y+1} of a permutation cycle σ_m^* of length

4 belong to C. Define $B = \{v_{ij} \in Even(\sigma_i^*) \ : \ 1 \leq i \leq s, \ i \neq m \ and \ 1 \leq j \leq p_i\}$. We note that $\sigma^{*^2}(B) = B$. Also $[v_{m2x}, B] \subseteq E(G)$ since v_{m2x} is a vertex in C not in B and $B \subseteq V'$. By Lemma2.2 $[\sigma^{*^2}(v_{m2x}), \sigma^{*^2}(B)] \subseteq E(G)$ since $[v_{m2x}, B] \subseteq E(G)$. So $[\sigma^{*^2}(v_{m2x}), B] \subseteq E(G)$ since $\sigma^{*^2}(B) = B$. The permutation cycle σ_m^* is of length 4 implies $\sigma^{*^2}(v_{m2x})$ is an even labelled vertex distinct from v_{m2x}. The vertices v_{m2x} and $\sigma^{*^2}(v_{m2x})$ are the only even labelled vertices of the permutation cycle σ_m^* implies $[v_{m2x}, \sigma^{*^2}(v_{m2x})] \in E(G)$ since σ^* is a star c.p. of G. We have $Even(\sigma^*) = B \bigcup \{v_{m2x}, \sigma^{*^2}(v_{m2x})\}$. Therefore $< Even(\sigma^*) >$ is a maximum clique of G since $[v_{m2x}, B] \subseteq E(G)$, $[\sigma^{*^2}(v_{m2x}), B] \subseteq E(G)$, $[v_{m2x}, \sigma^{*^2}(v_{m2x})] \in E(G)$, B is contained in the vertex set of C and $|Even(\sigma^*)| = 2n$. □

The following result is a sufficient condition for a s.c. graph with 4n vertices to be a split graph in terms of its clique number.

Theorem 2.15 : *Let G be a s.c. graph with p=4n and $\omega(G) = 2n$. Then G is a split graph.*

Proof: Let σ^* be a star c.p. of G. By Theorem 2.14 $< Even(\sigma^*) >$ is a maximum clique of G. We note $Odd(\sigma^*)$ is a stable set of G since $Odd(\sigma^*) = \sigma^*(Even(\sigma^*))$. Also the sets $Even(\sigma^*)$ and $Odd(\sigma^*)$ partition V(G). Hence G is a split graph. □

We obtain a sufficient condition for a s.c. graph with 4n+1 vertices to be a split graph in terms of its clique number.

Theorem 2.16 : *Let G be a s.c. graph with p=4n+1 and $\omega(G) = 2n+1$. Then G is a split graph.*

Proof: Let σ be a c.p. of G with v_0 as its fixed vertex. Let $G' = G - v_0$. By Corollary2.2 G' is a s.c. graph. Let C be a maximum clique of G. Let V_C be the vertex set of C. We prove that $v_0 \in V_C$. Let $v_0 \notin V_C$. We note that C is a maximum clique of G' since $v_0 \notin V_C$. Hence $\omega(G') = 2n + 1$, a contradiction to Theorem2.11. So $v_0 \in V_C$. Let $S = \sigma(V_C)$. We note that $\sigma(v_0) = v_0$ since v_0 is the fixed vertex of σ. Hence $v_0 \in S$ since $v_0 \in V_C$. Also S is a stable set of G since $< V_C >$ is a clique of G. We note that S and V_C have atmost one common element since $< V_C >$ is a clique of G and S is a stable set of G. Also $|S| = |V_C| = 2n + 1$. Hence the sets V_C and $S - \{v_0\}$ partition V(G). Therefore G is a split graph. □

The following result is a consolidation of Theorem2.15 and Theorem2.16.

Theorem 2.17 : *Let G be a s.c. graph. Then G is a split graph if*

(i) $\omega(G) = 2n$ *when p=4n*

(ii) $\omega(G) = 2n + 1$ *when p=4n+1.*

Proof: Follows from Theorem 2.15 and Theorem 2.16. □

A characterisation of a s.c. graph to be a split graph in terms of its clique number follows.

Theorem 2.18 : *Let G be a s.c. graph. Then G is a split graph if and only if*

(i) $\omega(G) = 2n$ *when p=4n*

(ii) $\omega(G) = 2n + 1$ *when p=4n+1.*

Proof: Follows from Theorem 2.10 and Theorem 2.17. □

Theorem 2.19 : *Let G be a s.c. graph. Then G is a split graph if and only if G is a chordal graph.*

Proof: Follows from Theorem 2.3. □

From the above result we see that the class of s.c. split graphs is exactly the class of s.c. chordal graphs. Hence we have the following characterisation for s.c. graphs to be chordal in terms of its clique number.

Theorem 2.20 : *Let G be a s.c. graph. Then G is a chordal graph if and only if*

(i) $\omega(G) = 2n$ *when p=4n*

(ii) $\omega(G) = 2n + 1$ *when p=4n+1.*

Proof: Follows from Theorem2.18 and Theorem2.19. □

2.4 A characterisation for self-complementary graphs to be chordal in terms of the stability number

The clique number of a graph is equal to the stability number of its complement.

Theorem 2.21 : *Let G be a graph. Then* $\omega(G) = \alpha(\bar{G})$.

Proof: Let $\omega(G) > \alpha(\bar{G})$. Let C be a maximum clique of G. Let V' be the vertex set of C. We note that V' is a stable set of $\bar{G}$ since C is a clique of G. So $\alpha(\bar{G}) \geq \omega(G)$ since $|V'| = \omega(G)$, a contradiction. Then $\omega(G) \leq \alpha(\bar{G})$. Let $\omega(G) < \alpha(\bar{G})$. Let S be a maximum stable set of $\bar{G}$. The set S induces a complete subgraph in G. Hence $\alpha(\bar{G}) \leq \omega(G)$ since $|S| = \alpha(\bar{G})$, a contradiction. Therefore $\omega(G) = \alpha(\bar{G})$. □

For a s.c. graph the clique number is equal to its stability number.

Theorem 2.22 : *Let G be a s.c. graph. Then* $\omega(G) = \alpha(G)$.

Proof: Follows from Theorem2.21 since G is a s.c. graph. □

The following result is a characterisation for a s.c. graph to be a chordal graph in terms of its stability number.

Theorem 2.23 : *Let G be a s.c. graph. Then G is a chordal graph if and only if*

(i) $\alpha(G) = 2n$ *when p=4n*

(ii) $\alpha(G) = 2n + 1$ *when p=4n+1.*

Proof: Follows from Theorem2.20 and Theorem2.22. □

2.5 A characterisation for self-complementary graphs to be chordal in terms of the induced cycles

Theorem 2.24 *Let G be a s.c. graph. Then G has no induced subgraph isomorphic to* C_4 *if and only if it has no induced subgraph isomorphic to* $2K_2$.

Proof: By Lemma2.3 G has no induced subgraph isomorphic to C_4 if and only if it has no induced subgraph isomorphic to $\bar{C_4} = 2K_2$. □

Lemma 2.7 : *Let G be a s.c. graph with p=4n and a c.p.* $\sigma = (1\ 2\ \cdots 4n)$. *Then* $[2, 2k+2] \in E(G)$ *if and only if* $[2, 4n+2-2k] \in E(G)$ *where k is an integer such that* $1 \leq k \leq n$.

Proof: By Lemma2.2 $[2, 2k+2] \in E(G)$ if and only if $[\sigma^{4n-2k}(2), \sigma^{4n-2k}(2k+2)] =$ $[4n+2-2k, 2] \in E(G)$ where $1 \leq k \leq n$. $\square$

Lemma 2.8 : *Let G be a s.c. graph with p=4n and a c.p.* $\sigma = (1\ 2\ \cdots 4n)$. *Then* $[2, 2k+2] \in E(G)$ *for all integers k such that* $n \leq k \leq 2n-1$ *if and only if* $[2, 2k+2] \in E(G)$ *for all integers k such that* $1 \leq k \leq n$.

Proof: Every vertex $2k+2$ where $n \leq k \leq 2n-1$ can be represented as $4n+2-2k$ for some integer k such that $1 \leq k \leq n$ and every vertex $4n+2-2k$ where $1 \leq k \leq n$ can be represented as $2k+2$ for some integer k such that $n \leq k \leq 2n-1$. So $[2, 2k+2] \in E(G)$ for all $n \leq k \leq 2n-1$ if and only if $[2, 4n+2-2k] \in E(G)$ for all $1 \leq k \leq n$. By Lemma2.7 $[2, 4n+2-2k] \in E(G)$ for all $1 \leq k \leq n$ if and only if $[2, 2k+2] \in E(G)$ for all $1 \leq k \leq n$. Hence $[2, 2k+2] \in E(G)$ for all $n \leq k \leq 2n-1$ if and only if $[2, 2k+2] \in E(G)$ for all $1 \leq k \leq n$. $\square$

Lemma 2.9 : *Let G be a s.c. graph with p=4n and a c.p.* $\sigma = (1\ 2\ \cdots 4n)$. *Let j be an integer such that* $1 \leq j \leq 2n-1$. *Then* $[2, 2k+2] \in E(G)$ *for all integers k such that* $1 \leq k \leq 2n-1$ *if and only if* $[2j+2, \sigma^{2k}(2j+2)] \in E(G)$ *for all integers k such that* $1 \leq k \leq 2n-1$.

Proof: By Lemma2.2 $[2, 2k+2] \in E(G)$ for all $1 \leq k \leq 2n-1$ if and only if $[\sigma^{2j}(2), \sigma^{2j}(2k+2)] = [2j+2, \sigma^{2k}(2j+2)] \in E(G)$ for all $1 \leq k \leq 2n-1$. $\square$

Lemma 2.10 : *Let G be a s.c. graph with p=8n and a c.p.* $\sigma = (1\ 2\ \cdots 8n)$. *Then* $[2, 4n+2] \in E(G)$ *and* $[2, 2n+2] \notin E(G)$ *implies G has an induced* C_4.

Proof: Let $[2, 4n+2] \in E(G)$ and $[2, 2n+2] \notin E(G)$. By Lemma2.2 $[\sigma^{2n}(2), \sigma^{2n}(4n+2)] =$ $[2n+2, 6n+2] \in E(G)$ since $[2, 4n+2] \in E(G)$. Also by Lemma2.2 $[\sigma^{2n}(2), \sigma^{2n}(2n+2)] =$ $[2n+2, 4n+2] \notin E(G)$, $[\sigma^{4n}(2), \sigma^{4n}(2n+2)] = [4n+2, 6n+2] \notin E(G)$ and $[\sigma^{6n}(2), \sigma^{6n}(2n+2)] = [6n+2, 2] \notin E(G)$ since $[2, 2n+2] \notin E(G)$. Hence the vertices 2, 2n+2, 4n+2 and 6n+2 induce a $2K_2$. Then by Theorem2.24 G has an induced C_4. $\square$

Lemma 2.11 : *Let G be a s.c. graph with p=4n and a star c.p.* $\sigma^* = (1\ 2\ \cdots 4n)$. *Then* $[2, 2n+2] \notin E(G)$ *implies that G has an induced* C_4.

Proof: Let $[2, 2n+2] \notin E(G)$. We note that $[2,4] \in E(G)$ since σ^* is a star c.p.. By Lemma2.2 $[\sigma^{*^{2n}}(2), \sigma^{*^{2n}}(4)] = [2n+2, 2n+4] \in E(G)$ since $[2,4] \in E(G)$. Also by Lemma2.2 $[\sigma^{*^2}(2), \sigma^{*^2}(2n+2)] = [4, 2n+4] \notin E(G)$ since $[2, 2n+2] \notin E(G)$.

Case i : Let $[4, 2n+2] \notin E(G)$. By Lemma2.2 $[\sigma^{*^{2n}}(4), \sigma^{*^{2n}}(2n+2)] = [2n+4, 2] \notin E(G)$ since $[4, 2n+2] \notin E(G)$. Hence the vertices 2, 4, 2n+2 and 2n+4 induce a $2K_2$. Then by Theorem2.24 G has an induced C_4.

Case ii : Let $[4, 2n+2] \in E(G)$. By Lemma2.2 $[\sigma^{*^{2n}}(4), \sigma^{*^{2n}}(2n+2)] = [2n+4, 2] \in E(G)$ since $[4, 2n+2] \in E(G)$. Hence the vertices 2, 4, 2n+2 and 2n+4 induce a C_4.

Therefore the Lemma. $\square$

Lemma 2.12 *: Let G be a s.c. graph with p=4n and a star c.p. $\sigma^* = (1\ 2 \cdots 4n)$. Then G is C_4-free implies that $< Even(\sigma^*) >$ is a complete subgraph of G.*

Proof: Let G be C_4-free. By Lemma2.9 for proving $< Even(\sigma^*) >$ is a complete subgraph of G, we have to prove that $[2, 2i+2] \in E(G)$ for all $1 \leq i \leq 2n-1$. By Lemma2.8 for proving $[2, 2i+2] \in E(G)$ for all $n \leq i \leq 2n-1$ we have to prove that $[2, 2i+2] \in E(G)$ for all $1 \leq i \leq n$. Hence for proving $< Even(\sigma^*) >$ is a complete subgraph of G we prove that $[2, 2i+2] \in E(G)$ for all $1 \leq i \leq n$. First we note that $[2, 2n+2] \in E(G)$. Otherwise by Lemma2.11 G has an induced C_4, a contradiction to the assumption that G is C_4-free. Now we prove that $[2, 2i+2] \in E(G)$ for all $1 \leq i \leq n-1$. Let $[2, 2k+2] \notin E(G)$ for an integer k such that $1 \leq k \leq n-1$. Consider the vertex $2n+2-2k$. Either $2n+2-2k = 2k+2$ or $2n+2-2k \neq 2k+2$. Let $2n+2-2k = 2k+2$. That is $n = 2k$. Then G is a s.c. graph with $p = 8k$ and a star c.p. $\sigma^* = (1\ 2 \cdots 8k)$. Also $[2, 4k+2] \in E(G)$ since $[2, 2n+2] \in E(G)$. By Lemma2.10 G has an induced C_4 since $[2, 2k+2] \notin E(G)$, a contradiction to the assumption that G is C_4-free. Therefore $2n+2-2k \neq 2k+2$. That is the vertices $2k+2$ and $2n+2-2k$ are distinct. By Lemma2.2 $[\sigma^{*^{2k}}(2), \sigma^{*^{2k}}(2n+2)] = [2k+2, 2n+2k+2] \in E(G)$ since $[2, 2n+2] \in E(G)$. Also by Lemma2.2 $[\sigma^{*^{2n}}(2), \sigma^{*^{2n}}(2k+2)] = [2n+2, 2n+2k+2] \notin E(G)$ since $[2, 2k+2] \notin E(G)$.

Case i : Let $[2, 2n+2-2k] \notin E(G)$. By Lemma2.2 $[\sigma^{*^{2k}}(2), \sigma^{*^{2k}}(2n+2-2k)] = [2k+2, 2n+2] \notin E(G)$ and $[\sigma^{*^{2n+2k}}(2), \sigma^{*^{2n+2k}}(2n+2-2k)] = [2n+2k+2, 2] \notin E(G)$ since $[2, 2n+2-2k] \notin E(G)$. Hence the vertices 2, 2k+2, 2r+2 and 2n+2k+2 induce a $2K_2$. Then

by Theorem2.24 G has an induced C_4, a contradiction to the assumption that G is C_4-free.
Case ii : Let $[2, 2n+2-2k] \in E(G)$. By Lemma2.2 $[\sigma^{*^{2k}}(2), \sigma^{*^{2k}}(2n+2-2k)] = [2k+2, 2n+2] \in E(G)$ and $[\sigma^{*^{2n+2k}}(2), \sigma^{*^{2n+2k}}(2n+2-2k)] = [2n+2k+2, 2] \in E(G)$ since $[2, 2n+2-2k] \in E(G)$. Hence the vertices 2, 2k+2, 2n+2 and 2n+2k+2 induce a C_4, a contradiction to the assumption that G is C_4-free.
Therefore $[2, 2i+2] \in E(G)$ for all $1 \leq i \leq n-1$. □

Lemma 2.13 : *Let G be a s.c. graph with p=4n and a c.p.* $\sigma = \sigma_1\sigma_2\cdots\sigma_s$ *where* $\sigma_i = (v_{i1}v_{i2}\cdots v_{ip_i})$ *for all* $1 \leq i \leq s$. *Then* $[Even(\sigma_i), Even(\sigma_j)] \subseteq E(G)$ *for all integers i and j such that* $1 \leq i \leq s$, $1 \leq j \leq s$ *and* $i \neq j$ *if and only if* $[v_{i2}, Even(\sigma_j)] \subseteq E(G)$ *for all integers i and j such that* $1 \leq i \leq s$, $1 \leq j \leq s$ *and* $i \neq j$.

Proof: By Theorem2.1 p_i is divisible by 4 for all $1 \leq i \leq s$. We note that $[v_{i2k}, Even(\sigma_j)] = [\sigma^{2k-2}(v_{i2}), \sigma^{2k-2}(Even(\sigma_j))] \subseteq E$ for all $1 \leq k \leq \frac{p_i}{2}$, $1 \leq i \leq s$, $1 \leq j \leq s$ and $i \neq j$ if and only if $[v_{i2}, Even(\sigma_j)] \subseteq E$ for all $1 \leq i \leq s$, $1 \leq j \leq s$ and $i \neq j$. □

For a s.c. graph G with 4n vertices and a star c.p. σ^* the collection of all even labelled vertices of σ^* induces a complete subgraph of G when G is C_4-free.

Theorem 2.25 : *Let G be a s.c. graph with p=4n and a star c.p.* σ^*. *Then G is* C_4*-free implies* $< Even(\sigma^*) >$ *is a complete subgraph of G.*

Proof: Let G be C_4-free. Let $\sigma^* = \sigma_1^*\sigma_2^*\cdots\sigma_s^*$ where $\sigma_i^* = (v_{i1}v_{i2}\cdots v_{ip_i})$ for all $1 \leq i \leq s$. Let $V_i = \{v_{ij} : 1 \leq j \leq p_i\}$ for all $1 \leq i \leq s$. By Theorem2.1 p_i is divisible by 4 for all $1 \leq i \leq s$. By Theorem2.13 $< V_i >$ is a s.c. graph with a star c.p. $(v_{i1}v_{i2}\cdots v_{ip_i})$ for all $1 \leq i \leq s$. Also $< V_i >$ is C_4-free for all $1 \leq i \leq s$ since G is C_4-free. Hence by Lemma2.12 $< Even(\sigma_i^*) >$ is a complete subgraph of $< V_i >$ where $1 \leq i \leq s$ and hence a complete subgraph of G. Then to prove that $< Even(\sigma^*) >$ is a complete subgraph of G we have to prove that $[Even(\sigma_i^*), Even(\sigma_j^*)] \subseteq E(G)$ for all $1 \leq i \leq s$, $1 \leq j \leq s$ and $i \neq j$. By Lemma2.13 to prove $[Even(\sigma_i^*), Even(\sigma_j^*)] \subseteq E(G)$ for all $1 \leq i \leq s$, $1 \leq j \leq s$ and $i \neq j$, we have to prove that $[v_{i2}, Even(\sigma_j^*)] \subseteq E(G)$ for all $1 \leq i \leq s$, $1 \leq j \leq s$ and $i \neq j$. Therefore to prove $< Even(\sigma^*) >$ is a complete subgraph of G we prove that $[v_{i2}, Even(\sigma_j^*)] \subseteq E(G)$ for all $1 \leq i \leq s$, $1 \leq j \leq s$ and $i \neq j$. Let $[v_{l2}, Even(\sigma_k^*)] \not\subseteq E(G)$ for two distinct integers

l and k such that $1 \leq l \leq s$ and $1 \leq k \leq s$. There exists a vertex v_{k2x} of $Even(\sigma_k^*)$ such that $[v_{l2}, v_{k2x}] \notin E(G)$ since $[v_{l2}, Even(\sigma_k^*)] \nsubseteq E(G)$. We prove that there exists a vertex v_{k2y} of $Even(\sigma_k^*)$ such that $[v_{l2}, v_{k2y}] \in E(G)$. Otherwise $[v_{l2}, v_{k2i}] \notin E(G)$ for all $v_{k2i} \in Even(\sigma_k^*)$. We note that $[v_{l2}, v_{k2i}] \notin E(G)$ for all $v_{k2i} \in Even(\sigma_k^*)$ implies $[v_{l2}, v_{k2}] \notin E(G)$ and $[v_{l2}, v_{k4}] \notin E(G)$. By Lemma2.2 $[\sigma^{*^2}(v_{l2}), \sigma^{*^2}(v_{k2i})] = [v_{l4}, \sigma^{*^2}(v_{k2i})] \notin E(G)$ for all $v_{k2i} \in Even(\sigma_k^*)$ since $[v_{l2}, v_{k2i}] \notin E(G)$ for all $v_{k2i} \in Even(\sigma_k^*)$. Then $[v_{l4}, v_{k2}] \notin E(G)$ and $[v_{l4}, v_{k4}] \notin E(G)$. We have $[v_{l2}, v_{l4}] \in E(G)$ and $[v_{k2}, v_{k4}] \in E(G)$ since $< Even(\sigma_l^*) >$ and $< Even(\sigma_k^*) >$ are complete subgraphs of G. Therefore the vertices v_{l2}, v_{l4}, v_{k2} and v_{k4} induces a $2K_2$. Then by Theorem2.24 G has an induced C_4, a contradiction to the assumption that G is C_4-free. Hence there exists a vertex v_{k2y} of $Even(\sigma_k^*)$ such that $[v_{l2}, v_{k2y}] \in E(G)$. We note that one of the following cases should hold.

(a) $[v_{l2}, v_{k2}] \in E(G)$ and $[v_{l4}, v_{k2}] \notin E(G)$

(b) $[v_{l2}, v_{k2}] \notin E(G)$ and $[v_{l4}, v_{k2}] \in E(G)$

(c) $[v_{l2}, v_{k2}] \in E(G)$ and $[v_{l4}, v_{k2}] \in E(G)$

(d) $[v_{l2}, v_{k2}] \notin E(G)$ and $[v_{l4}, v_{k2}] \notin E(G)$.

We prove that the cases (a), (b), (c) and (d) are impossible.

(a) Choose the vertex v_{k2m} of $Even(\sigma_k^*)$ such that $[v_{l2}, v_{k2m}] \notin E(G)$ and $[v_{l2}, v_{k2i}] \in E(G)$ for all $1 \leq i \leq m-1$ since $[v_{l2}, v_{k2x}] \notin E(G)$ the vertex v_{k2m} exists. By Lemma2.2 $[\sigma^{*^2}(v_{l2}), \sigma^{*^2}(v_{k2(m-1)})] = [v_{l4}, v_{k2m}] \in E(G)$ since $[v_{l2}, v_{k2(m-1)}] \in E(G)$. Also $[v_{l2}, v_{l4}] \in E(G)$ and $[v_{k2}, v_{k2m}] \in E(G)$ since $< Even(\sigma_l^*) >$ and $< Even(\sigma_k^*) >$ are complete subgraphs of G. Hence the vertices v_{l2}, v_{l4}, v_{k2} and v_{k2m} induce a C_4, a contradiction to the assumption that G is C_4-free. Therefore this case is not possible.

(b) Choose the vertex v_{k2m} of $Even(\sigma_k^*)$ such that $[v_{l2}, v_{k2m}] \in E(G)$ and $[v_{l2}, v_{k2i}] \notin E(G)$ for all $1 \leq i \leq m-1$, since $[v_{l2}, v_{k2y}] \in E(G)$ the vertex v_{k2m} exists. By Lemma2.2 $[\sigma^{*^2}(v_{l2}), \sigma^{*^2}(v_{k2(m-1)})] = [v_{l4}, v_{k2m}] \notin E(G)$ since $[v_{l2}, v_{k2(m-1)}] \notin E(G)$. Also $[v_{l2}, v_{l4}] \in E(G)$ and $[v_{k2}, v_{k2m}] \in E(G)$ since $< Even(\sigma_l^*) >$ and $< Even(\sigma_k^*) >$ are complete subgraphs of G. Hence the vertices v_{l2}, v_{l4}, v_{k2} and v_{k2m} induce a C_4, a contradiction to the assumption that G is C_4-free. Therefore this case is not possible.

(c) Choose the vertex $v_{k2m} \in Even(\sigma_k^*)$ such that $[v_{l2}, v_{k2m}] \notin E(G)$ and $[v_{l2}, v_{k2i}] \in E(G)$ for all $1 \leq i \leq m-1$ since $[v_{l2}, v_{k2x}] \notin E(G)$ the vertex v_{k2m} exists. By Lemma2.2 $[\sigma^{*^2}(v_{l2}), \sigma^{*^2}(v_{k2(m-1)})] = [v_{l4}, v_{k2m}] \in E(G)$ since $[v_{l2}, v_{k2(m-1)}] \in E(G)$. Now choose the vertex $\sigma^{*^{2r}}(v_{k2m}) \in Even(\sigma_k^*)$ such that $[v_{l2}, \sigma^{*^{2r}}(v_{k2m})] \in E(G)$ and $[v_{l2}, \sigma^{*^{2i}}(v_{k2m})] \notin E(G)$ for all $0 \leq i \leq r-1$, since $[v_{l2}, v_{k2}] \in E(G)$ the vertex $\sigma^{*^{2r}}(v_{k2m})$ exists. By Lemma2.2 $[\sigma^{*^2}(v_{l2}), \sigma^{*^{2r}}(v_{k2m})] = [v_{l4}, \sigma^{*^{2r}}(v_{k2m})] \notin E(G)$ since $[v_{l2}, \sigma^{*^{2(r-1)}}(v_{k2m})] \notin E(G)$. Also $[v_{l2}, v_{l4}] \in E(G)$ and $[v_{k2m}, \sigma^{*^{2r}}(v_{k2m})] \in E(G)$ since $< Even(\sigma_l^*) >$ and $< Even(\sigma_k^*) >$ are complete subgraphs of G. Hence the vertices v_{l2}, v_{l4}, v_{k2m} and $\sigma^{*^{2r}}(v_{k2m})$ induce a C_4, a contradiction to the assumption that G is C_4-free. Therefore this case is not possible.

(d) Choose the vertex $v_{k2m} \in Even(\sigma_k^*)$ such that $[v_{l2}, v_{k2m}] \in E(G)$ and $[v_{l2}, v_{k2i}] \notin E(G)$ for all $1 \leq i \leq m-1$ since $[v_{l2}, v_{k2y}] \in E(G)$ the vertex v_{k2m} exists. By Lemma2.2 $[\sigma^{*^2}(v_{l2}), \sigma^{*^2}(v_{k2(m-1)})] = [v_{l4}, v_{k2m}] \notin E(G)$ since $[v_{l2}, v_{k2(m-1)}] \notin E(G)$. Now choose the vertex $\sigma^{*^{2r}}(v_{k2m}) \in Even(\sigma_k^*)$ such that $[v_{l2}, \sigma^{*^{2r}}(v_{k2m})] \notin E(G)$ and $[v_{l2}, \sigma^{*^{2i}}(v_{k2m})] \in E(G)$ for all $0 \leq i \leq r-1$, since $[v_{l2}, v_{k2}] \notin E(G)$ the vertex $\sigma^{*^{2r}}(v_{k2m})$ exists. By Lemma2.2 $[\sigma^{*^2}(v_{l2}), \sigma^{*^{2r}}(v_{k2m})] = [v_{l4}, \sigma^{*^{2r}}(v_{k2m})] \in E(G)$ since $[v_{l2}, \sigma^{*^{2(r-1)}}(v_{k2m})] \in E(G)$. Also $[v_{l2}, v_{l4}] \in E(G)$ and $[v_{k2m}, \sigma^{*^{2r}}(v_{k2m})] \in E(G)$ since $< Even(\sigma_l^*) >$ and $< Even(\sigma_k^*) >$ are complete subgraphs of G. Hence the vertices v_{l2}, v_{l4}, v_{k2m} and $\sigma^{*^{2r}}(v_{k2m})$ induce a C_4, a contradiction to the assumption that G is C_4-free. Therefore this case is not possible.

So the cases (a), (b), (c) and (d) are impossible, a contradiction to the fact that either one of them should hold. Therefore $[v_{i2}, Even(\sigma_j^*)] \subseteq E(G)$ for $1 \leq i \leq s$, $1 \leq j \leq s$ and $i \neq j$. □

Every C_4-free s.c. graph with 4n vertices is a split graph.

Theorem 2.26 : *Let G be a s.c. graph with p=4n. Then G is C_4-free implies that G is a split graph.*

Proof: Let G be C_4-free. Let σ^* be a star c.p. of G. By Theorem2.25 $< Even(\sigma^*) >$ is a complete subgraph of G. We note that $Odd(\sigma^*)$ is a stable set of G since $Odd(\sigma^*) = \sigma^*(Even(\sigma^*))$. Hence G is a split graph since the sets $Even(\sigma^*)$ and $Odd(\sigma^*)$ partition V(G).□

Theorem 2.27 : *Let G be a s.c. graph with p=4n. Then G is split graph implies that G has no induced subgraph isomorphic to C_5.*

Proof: Follows from Theorem2.2. □

Every s.c. graph with 4n vertices having an induced subgraph isomorphic to C_5 has an induced subgraph isomorphic to C_4.

Theorem 2.28 : *Let G be a s.c. graph with p=4n. Then G has no induced subgraph isomorphic to C_4 implies that G has no induced subgraph isomorphic to C_5.*

Proof: Let G be a s.c. graph with 4n vertices which has no induced subgraph isomorphic to C_4 and has an induced subgraph isomorphic to C_5. By Theorem2.26 G is a split graph since G is C_4-free. Then by Theorem2.27 G has no induced subgraph isomorphic to C_5 since G is a split graph, a contradiction. □

However, in the case of a s.c. graph G with 4n+1 vertices G has no induced subgraph isomorphic to C_4 is not sufficient to deduce that G has no induced subgraph isomorphic to C_5. For example the s.c. graph C_5 has no induced subgraph isomorphic to C_4.

Theorem 2.29 : *Let G be a s.c. graph with p=4n. Then G has no induced subgraph isomorphic to C_4 implies that G has no induced subgraph isomorphic to $2K_2$ or C_5.*

Proof: Follows from Theorem2.24 and Theorem2.28. □

The following result is a characterisation for a s.c. graph to be a split graph in terms of its induced cycles.

Theorem 2.30 : *Let G be a s.c. graph. Then G is a split graph if and only if G has no induced subgraph isomorphic to C_4 or C_5.*

Proof: Follows from Theorem2.2 and Theorem2.24. □

We have the following characterisation for s.c. graphs to be split graphs in terms of its induced cycles which is a modification of Theorem2.30 for the case of s.c. graph with 4n vertices.

Theorem 2.31 : *Let G be a s.c. graph. Then G is a split graph if and only if*

(i) *G has no induced subgraph isomorphic to* C_4 *when p=4n*

(ii) *G has no induced subgraph isomorphic to* C_4 *or* C_5 *when p=4n+1.*

Proof: (i) Follows from Theorem2.2 and Theorem2.29.
(ii) Follows from Theorem2.30. □

The following result is a characterisation for a s.c. graph to be a chordal graph in terms of its induced cycles.

Theorem 2.32 : *Let G be a s.c. graph. Then G is a chordal graph if and only if*

(i) *G has no induced subgraph isomorphic to* C_4 *when p=4n*

(ii) *G has no induced subgraph isomorphic to* C_4 *or* C_5 *when p=4n+1.*

Proof: Follows from Theorem2.19 and Theorem2.31. □

2.6 A characterisation for self-complementary graphs to be chordal in terms of the degree sequence

Hammer and Simone [109] obtained a characterisation for split graphs in terms of its degree sequence. We obtain a characterisation for s.c. graphs to be split graphs in terms of its degree sequence.

Theorem 2.33 *Let G be a s.c. graph with degree sequence* $d_1 \geq d_2 \geq \cdots \geq d_p$. *Then G is a split graph if and only if*

(i) $\sum_{i=1}^{2n} d_i = 6n^2 - 2n$ *when p=4n*

(ii) $\sum_{i=1}^{2n} d_i = 6n^2$ *when p=4n+1.*

Proof: (i) Let p=4n. Let $\mathcal{M} = max\{i \ : \ d_i \geq i-1, \ 1 \leq i \leq p\}$. By Theorem2.9 $\mathcal{M} = 2n$. By Theorem2.8 $d_i = 4n - 1 - d_{4n-i+1}$ for all $2n+1 \leq i \leq 4n$. Then $\sum_{i=2n+1}^{4n} d_i = 8n^2 - 2n - \sum_{i=1}^{2n} d_i$. Hence by Theorem2.5 the graph G is a split graph if and only if $\sum_{i=1}^{2n} d_i = 6n^2 - 2n$.

(ii) Let p=4n+1. Let $\mathcal{M} = max\{i : d_i \geq i-1, 1 \leq i \leq p\}$. By Theorem2.9 $\mathcal{M} = 2n+1$. By Theorem2.8 $d_i = 4n - d_{4n-i+2}$ for all $2n+2 \leq i \leq 4n+1$. Then $\sum_{i=2n+2}^{4n+1} d_i = 8n^2 - \sum_{i=1}^{2n} d_i$. Also by Theorem2.8 $d_{2n+1} = 2n$. Hence by Theorem2.5 the graph G is a split graph if and only if $\sum_{i=1}^{2n} d_i = 6n^2$. □

A characterisation for a s.c. graph to be a chordal graph in terms of its degree sequence follows.

Theorem 2.34 *Let G be a s.c. graph with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_p$. Then G is a chordal graph if and only if*

(i) $\sum_{i=1}^{2n} d_i = 6n^2 - 2n$ *when p=4n*

(ii) $\sum_{i=1}^{2n} d_i = 6n^2$ *when p=4n+1.*

Proof: Follows from Theorem2.19 and Theorem2.33. □

Chapter 3

On the existence and the construction of self-complementary chordal graphs

3.1 Introduction

Existence problems of self-complementary graphs had been studied by Ringel[178] and Sachs[190]. They prove that a s.c. graph with p vertices exists if and only if p=4n or p=4n+1 for some positive integer n. In Section 3.3 we prove that a s.c. chordal graph with p vertices exists if and only if p=4n or p=4n+1 for some positive integer n.

Gibbs[98] has given algorithms for the construction of all s.c. graphs with 4n vertices and 4n+1 vertices. We discuss these algorithms in Section 3.2. In Section 3.4 by modifying these algorithms we give algorithms for the construction of all s.c. chordal graphs with 4n vertices and 4n+1 vertices.

3.2 Algorithms for constructing self-complementary graphs

The following algorithm was given by Gibbs [98] for constructing all s.c. graphs with 4n vertices.

Algorithm 3.1 (Gibbs [98]) : An algorithm to construct s.c. graphs with 4n vertices where n is a positive integer

Choose positive integers $p_1, p_2, \ldots, p_s$ such that $\sum_{i=1}^{s} p_i = 4n$, $p_i \leq p_j$ for all $1 \leq i < j \leq s$ and $p_i = 2^{k_i}$ for some $k_i \geq 2$ where $1 \leq i \leq s$. Let $\sigma = \sigma_1\sigma_2 \cdots \sigma_s$ where $\sigma_i = (v_{i1}v_{i2} \ldots v_{ip_i})$ for all $1 \leq i \leq s$ be a permutation on 4n labels $v_{11}, v_{12}, \ldots, v_{1p_1}$, $v_{21}, v_{22}, \ldots, v_{2p_2}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_s}$. For the label v_{i1} where $1 \leq i \leq s-1$ define $v_{i2}, v_{i3}, \ldots, v_{i(\frac{p_i}{2}+1)}$, $v_{(i+1)1}, v_{(i+1)2}, \ldots, v_{(i+1)p_i}, v_{(i+2)1}, v_{(i+2)2}, \ldots, v_{(i+2)p_i}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_i}$ as the range of v_{i1}. For the label v_{s1} define $v_{s2}, v_{s3}, \ldots, v_{s(\frac{p_s}{2}+1)}$ as its range. Construct a graph G with 4n vertices by identifying the vertices of G with the labels v_{ij} where $1 \leq i \leq s$ and $1 \leq j \leq p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_{i1}, v_{jr}]$ where $1 \leq i \leq s$ and v_{jr} is a vertex in the range of v_{i1} it is arbitrarily decided whether $[v_{i1}, v_{jr}] \in E(G)$ or $[v_{i1}, v_{jr}] \notin E(G)$.

(b) After completing (a) whether the pairs $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ or $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ for all $1 \leq k \leq p_j - 1$ is determined as follows for each pair $[v_{i1}, v_{jr}]$ such that $1 \leq i \leq s$ and v_{jr} is in the range of v_{i1}. The pair $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is even and the pair $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is odd.

The following Theorem states that a graph G is a s.c. graph with 4n vertices if and only if it can be constructed by Algorithm3.1.

Theorem 3.1 *(Gibbs [98]) : (i) The graphs constructed by Algorithm3.1 are s.c. graphs with 4n vertices where n is a positive integer.*

(ii) Every s.c. graph with 4n vertices for all positive integer n can be constructed by Algorithm3.1.

We illustrate Algorithm3.1 by constructing a s.c. graph with 12 vertices. Two positive integers $p_1 = 4$ and $p_2 = 8$ are chosen. We note that the positive integers p_1 and p_2 are are such that $p_1 + p_2 = 12$, $p_1 \leq p_2$, $p_1 = 2^2$ and $p_2 = 2^3$. Let $\sigma = (v_{11}v_{12}v_{13}v_{14})(v_{21}v_{22}v_{23}v_{24}v_{25}v_{26}v_{27}v_{28})$ be a permutation on 12 labels v_{ij} where $i = 1, 2$ and $1 \leq j \leq p_i$. For the label v_{11} the labels $v_{12}, v_{13}, v_{21}, v_{22}, v_{23}$ and v_{24} constitute its range. For the label v_{21} the labels v_{22}, v_{23}, v_{24} and v_{25} constitute its range. We construct a graph G with 12 vertices by identifying the vertices of G with the labels v_{ij} where $i = 1, 2$ and $1 \leq j \leq p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_{i1}, v_{jr}]$ where $i = 1, 2$ and v_{jr} is a vertex in the range of v_{i1} we decide arbitrarily whether $[v_{i1}, v_{jr}] \in E(G)$ or $[v_{i1}, v_{jr}] \notin E(G)$. Let the vertex v_{11} be adjacent to the vertices v_{22}, v_{23} and v_{24} and non-adjacent to the vertices v_{12}, v_{13} and v_{21} of its range as shown in Figure 3.1(a). Let the vertex v_{21} be adjacent to the vertices v_{22}, v_{23} and v_{25} and non-adjacent to the vertex v_{24} of its range as shown in Figure 3.1(a).
(b) After completing (a) whether $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ or $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ for all $1 \leq k \leq p_j - 1$ is determined as follows for each pair $[v_{i1}, v_{jr}]$ where $i = 1, 2$ and v_{jr} is in the range of v_{i1}. The pair $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is even and $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is odd.

The labelled graph G constructed is shown in Figure 3.1(b). We note that σ is a c.p. of the s.c. graph G.

The following algorithm was given by Gibbs [98] for the construction of all s.c. graphs with 4n+1 vertices.

Algorithm 3.2 (Gibbs [98]) : An algorithm to construct s.c. graphs with 4n+1 vertices where n is a positive integer

Choose positive integers $p_2, p_3, \ldots, p_s$ such that $\sum_{i=2}^{s} p_i = 4n$, $p_i \leq p_j$ for all $2 \leq i < j \leq s$ and $p_i = 2^{k_i}$ for some $k_i \geq 2$ where $2 \leq i \leq s$. Let $\sigma = \sigma_1\sigma_2 \cdots \sigma_s$ where $\sigma_1 = (v_0)$ and $\sigma_i = (v_{i1}v_{i2} \ldots v_{ip_i})$ for all $2 \leq i \leq s$ be a permutation on 4n+1 labels $v_0, v_{21}, v_{22}, \ldots, v_{2p_2}, v_{31}, v_{32}, \ldots, v_{3p_3}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_s}$. For the label v_0 define $v_{21}, v_{31}, \ldots, v_{s1}$ as its range. For the label v_{i1} where $2 \leq i \leq s-1$ define $v_{i2}, v_{i3}, \ldots, v_{i(\frac{p_i}{2}+1)}, v_{(i+1)1}, v_{(i+1)2}, \ldots, v_{(i+1)p_i}, v_{(i+2)1}, v_{(i+2)2}, \ldots, v_{(i+2)p_i}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_i}$ as the range of v_{i1}. For the label v_{s1} define $v_{s2}, v_{s3}, \ldots, v_{s(\frac{p_s}{2}+1)}$ as its range. Construct a graph G with 4n+1 vertices by identifying the vertices of G with the labels v_0 and v_{ij} where $2 \leq i \leq s$, $1 \leq j \leq p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_0, v_{i1}]$ where v_{i1} is in the range of v_0 it is arbitrarily decided whether $[v_0, v_{i1}] \in E(G)$ or $[v_0, v_{i1}] \notin E(G)$.

(b) After completing (a) whether the pairs $[\sigma^k(v_0), \sigma^k(v_{i1})] \in E(G)$ or $[\sigma^k(v_0), \sigma^k(v_{i1})] \notin E(G)$ for all $1 \leq k \leq p_i - 1$ is determined as follows for each pair $[v_0, v_{i1}]$ such that v_{i1} is in the range of v_0. The pair $[\sigma^k(v_0), \sigma^k(v_{i1})] \in E(G)$ if and only if $[v_0, v_{i1}] \in E(G)$ when k is

even and $[\sigma^k(v_0), \sigma^k(v_{i1})] \notin E(G)$ *if and only if* $[v_0, v_{i1}] \in E(G)$ *when k is odd.*

(c) For each pair $[v_{i1}, v_{jr}]$ *where* $2 \leq i \leq s$ *and* v_{jr} *is a vertex in the range of* v_{i1} *it is arbitrarily decided whether* $[v_{i1}, v_{jr}] \in E(G)$ *or* $[v_{i1}, v_{jr}] \notin E(G)$.

(d) After completing (c) whether the pairs $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ *or* $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ *for all* $1 \leq k \leq p_j - 1$ *is determined as follows for each pair* $[v_{i1}, v_{jr}]$ *such that* $2 \leq i \leq s$ *and* v_{jr} *is in the range of* v_{i1}*. The pair* $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ *if and only if* $[v_{i1}, v_{jr}] \in E(G)$ *when k is even and the pair* $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ *if and only if* $[v_{i1}, v_{jr}] \in E(G)$ *when k is odd.*

The following Theorem states that a graph G is a s.c. graph with 4n+1 vertices if and only if it can be constructed by Algorithm 3.2.

Theorem 3.2 *(Gibbs [98]) : (i) The graphs constructed by Algorithm3.2 are s.c. graphs with 4n+1 vertices where n is a positive integer.*

(ii) Every s.c. graph with 4n+1 vertices for all positive integer n can be constructed by Algorithm3.2.

We illustrate Algorithm3.2 by constructing a s.c. graph with 13 vertices. Two positive integers $p_2 = 4$ and $p_3 = 8$ are chosen. We note that the positive integers p_2 and p_3 are such that $p_2+p_3 = 12$, $p_2 \leq p_3$, $p_2 = 2^2$ and $p_3 = 2^3$. Let $\sigma = (v_0)(v_{21}v_{22}v_{23}v_{24})(v_{31}v_{32}v_{33}v_{34}v_{35}v_{36}v_{37}v_{38})$ be a permutation on 13 labels v_0 and v_{ij} where $i = 2, 3,\ 1 \leq j \leq p_i$. For the label v_0 the labels v_{21} and v_{31} constitute its range. For the label v_{21} the labels $v_{22}, v_{23}, v_{31}, v_{32}, v_{33}$ and v_{34} constitute its range. For the label v_{31} the labels v_{32}, v_{33}, v_{34} and v_{35} constitute its range. We construct a graph G with 13 vertices by identifying the vertices of G with the labels v_0 and v_{ij} where $i = 2, 3,\ 1 \leq j \leq p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_0, v_{i1}]$ where v_{i1} is in the range of v_0 we decide arbitrarily whether $[v_0, v_{i1}] \in E(G)$ or $[v_0, v_{i1}] \notin E(G)$. Let the vertex v_0 be adjacent to v_{31} of its range and non-adjacent to v_{21} of its range as shown in Figure 3.2(a).

(b) After completing (a) whether the pair $[\sigma^k(v_0), \sigma^k(v_{i1})] \in E(G)$ or $[\sigma^k(v_0), \sigma^k(v_{i1})] \notin E(G)$ for all $1 \leq k \leq p_i - 1$ is determined as follows for each pair $[v_0, v_{i1}]$ such that v_{i1} is in the

range of v_0. The pair $[\sigma^k(v_0), \sigma^k(v_{i1})] \in E(G)$ if and only if $[v_0, v_{i1}] \in E(G)$ when k is even and $[\sigma^k(v_0), \sigma^k(v_{i1})] \notin E(G)$ if and only if $[v_0, v_{i1}] \in E(G)$ when k is odd.

(c) For each pair $[v_{i1}, v_{jr}]$ where $i = 2, 3$ and v_{jr} is a vertex in the range of v_{i1}, we decide arbitrarily whether $[v_{i1}, v_{jr}] \in E(G)$ or $[v_{i1}, v_{jr}] \notin E(G)$. Let the vertex v_{21} be adjacent to the vertices v_{32}, v_{33} and v_{34} and non-adjacent to the vertices v_{22}, v_{23} and v_{31} of its range as shown in Figure 3.2(a). Let the vertex v_{31} be adjacent to the vertices v_{32}, v_{33} and v_{35} and non-adjacent to the vertex v_{34} of its range as shown in Figure 3.2(a).

(d) After completing (c) whether $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ or $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ for all $1 \leq k \leq p_j - 1$ is determined as follows for each pair $[v_{i1}, v_{jr}]$ where $i = 2, 3$ and v_{jr} is in the range of v_{i1}. The pair $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \in E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is even and $[\sigma^k(v_{i1}), \sigma^k(v_{jr})] \notin E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is odd.

The labelled graph G constructed is shown in Figure 3.2(b). We note that σ is a c.p. of the s.c. graph G.

3.3 On the existence of self-complementary chordal graphs

The following result was obtained independently by Ringel [178] and Sachs [190] for the existence of s.c. graphs.

Theorem 3.3 *(Ringel [178], Sachs [190]) : A self-complementary graph with p vertices exists if and only if p=4n or p=4n+1 where n is a positive integer.*

The following result gives a necessary condition for the existence of s.c. chordal graphs.

Theorem 3.4 *: A s.c. chordal graph with p vertices exists only if p=4n or p=4n+1 where n is a positive integer.*

Proof: Follows from Theorem3.3. □

Existence of s.c. chordal graphs with 4n vertices for all positive integers n is assured by the following Theorem.

Theorem 3.5 *: There exists s.c. chordal graphs with 4n vertices for all positive integer n.*

Proof: Let n be a positive integer. Let G be a graph with 4n vertices $0, 1, \ldots, 4n-1$ whose adjacencies are defined by the following conditions (a),(b) and (c).

(a) Any two even labelled vertices are adjacent.

(b) Any two odd labelled vertices are non-adjacent.

(c) The vertex $2i+1$ where $0 \leq i \leq 2n-1$ is adjacent to the vertices $(2i+2)_{4n}, (2i+4)_{4n}, \ldots,$ $(2i+2n)_{4n}$ and non-adjacent to the vertices $(2i+2n+2)_{4n}, (2i+2n+4)_{4n}, \ldots,$ $(2i+4n)_{4n}$ where for a non-negative integer k the non-negative integer $(k)_{4n}$ is such that $k \equiv (k)_{4n} \pmod{4n}$.

The graphs G when n=1, n=2 and n=3 are shown in Figure 3.3(a), Figure 3.3(b) and Figure 3.3(c) respectively.

The graph G is a s.c. graph since $(0\ 1\ 2\ \cdots 4n-1)$ is a c.p. of G. Let $V_1 = \{0, 2, 4, \ldots, 4n-2\}$ and $V_2 = \{1, 3, \ldots, 4n-1\}$. We note that $< V_1 >$ is a complete subgraph of G and V_2 is a stable set of G. Also the sets V_1 and V_2 partition $V(G)$. Hence G is a split graph. So by Theorem2.19 G is a s.c. chordal graph. □

The existence of s.c. chordal graphs with 4n+1 vertices for all positive integer n is assured by the following Theorem.

Theorem 3.6 : *There exist s.c. chordal graphs with 4n+1 vertices for all positive integers n.*

Proof: Let n be a positive integer. Let G be a graph with 4n+1 vertices $0, 1, 2, \ldots, 4n$ whose adjacencies are defined by the following conditions (a), (b), (c) and (d).

(a) Any two even labelled vertices are adjacent.

(b) Any two odd labelled vertices are non-adjacent.

(c) The vertex 4n is non-adjacent to the vertices $1, 3, \ldots, 4n-1$.

(d) The vertex $2i+1$ where $0 \leq i \leq 2n-1$ is adjacent to the vertices $(2i+2)_{4n}, (2i+4)_{4n}, \ldots,$ $(2i+2n)_{4n}$ and non-adjacent to the vertices $(2i+2n+2)_{4n}, (2i+2n+4)_{4n}, \ldots,$ $(2i+4n)_{4n}$ where for a non-negative integer k the non-negative integer $(k)_{4n}$ is such that $k \equiv (k)_{4n} \pmod{4n}$.

The graphs G when n=1, n=2 and n=3 are shown in Figure 3.4(a), Figure 3.4(b) and Figure 3.4(c).

The graph G is a s.c. graph since $(0\ 1\ 2 \cdots 4n-1)(4n)$ is a c.p. of G. Let $V_1 = \{0, 2, 4, \ldots, 4n\}$ and $V_2 = \{1, 3, \ldots, 4n-1\}$. We note that $< V_1 >$ is a complete subgraph of G and V_2 is a stable set of G. Also V_1 and V_2 partition $V(G)$. Hence G is a split graph. Therefore by Theorem2.19 G is a chordal graph. □

The following result gives a necessary and sufficient condition for the existence of a s.c. chordal graph.

Theorem 3.7 : *A s.c. chordal graph with p vertices exists if and only if p=4n or p=4n+1 where n is a positive integer.*

Proof: Follows from Theorem3.4, Theorem3.5 and Theorem3.6. □

3.4 Algorithms for constructing self-complementary chordal graphs

The following algorithm constructs all s.c. chordal graphs with 4n vertices.

Algorithm 3.3 : An algorithm to construct s.c. chordal graphs with 4n vertices where n is a positive integer

Choose positive integers $p_1, p_2, \ldots, p_s$ such that $\sum_{i=1}^{s} p_i = 4n$, $p_i \le p_j$ for all $1 \le i < j \le s$ and $p_i = 2^{k_i}$ for some $k_i \ge 2$ where $1 \le i \le s$. Let $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ where $\sigma_i^* = (v_{i1} v_{i2} \ldots v_{ip_i})$ for all $1 \le i \le s$ be a permutation on 4n labels $v_{11}, v_{12}, \ldots, v_{1p_1}, v_{21}, v_{22}, \ldots, v_{2p_2}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_s}$. For the label v_{i1} where $1 \le i \le s-1$ define $v_{i2}, v_{i3}, \ldots, v_{i(\frac{p_i}{2}+1)}$, $v_{(i+1)1}, v_{(i+1)2}, \ldots, v_{(i+1)p_i}, v_{(i+2)1}, v_{(i+2)2}, \ldots, v_{(i+2)p_i}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_i}$ as the range of v_{i1}. For the label v_{s1} define $v_{s2}, v_{s3}, \ldots, v_{s(\frac{p_s}{2}+1)}$ as its range. Construct a graph G with 4n vertices by identifying the vertices of G with the labels v_{ij} where $1 \le i \le s$ and $1 \le j \le p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_{i1}, v_{jr}]$ where $1 \le i \le s$ and v_{jr} is a vertex in the range of v_{i1} the pair $[v_{i1}, v_{jr}] \notin E(G)$ when r is odd and it is arbitrarily decided whether $[v_{i1}, v_{jr}] \in E(G)$ or $[v_{i1}, v_{jr}] \notin E(G)$ when r is even.

(b) After completing (a) whether the pairs $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ or $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin$

$E(G)$ for all $1 \leq k \leq p_j - 1$ is determined as follows for each pair $[v_{i1}, v_{jr}]$ such that $1 \leq i \leq s$ and v_{jr} is in the range of v_{i1}. The pair $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is even and the pair $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is odd.

The following Theorem is a characterisation of a s.c. graph G with 4n vertices to be a chordal graph in terms of its induced subgraph $< Even(\sigma^*) >$ where σ^* is a star c.p. of G.

Theorem 3.8 : *Let G be a s.c. graph with p=4n and a star c.p. σ^*. Then G is a chordal graph if and only if $< Even(\sigma^*) >$ is a complete subgraph of G.*

Proof: Let G be a chordal graph. By Theorem2.20 $\omega(G) = 2n$. Hence by Theorem2.14 $< Even(\sigma^*) >$ is a complete subgraph of G.

Let $< Even(\sigma^*) >$ be a complete subgraph of G. We note that $\omega(G) \geq 2n$ since $|Even(\sigma^*)| = 2n$. Hence by Theorem2.11 $\omega(G) = 2n$. By Theorem2.20 it follows that G is a chordal graph. □

The following Theorem states that a graph G with 4n vertices is a s.c. chordal graph if and only if it can be constructed by Algorithm3.3.

Theorem 3.9 : *(i) The graphs constructed by Algorithm3.3 are s.c. chordal graphs with 4n vertices where n is a positive integer.*

(ii) Every s.c. chordal graph with 4n vertices for all positive integers n can be constructed by Algorithm3.3.

Proof: (i) Let n be a positive integer. Let $p_1, p_2, \ldots, p_s$ be positive integers such that $\sum_{i=1}^{s} p_i = 4n$, $p_i \leq p_j$ for all $1 \leq i < j \leq s$ and $p_i = 2^{k_i}$ for some $k_i \geq 2$ where $1 \leq i \leq s$. Let $\sigma^* = \sigma_1^* \sigma_2^* \ldots \sigma_s^*$ where $\sigma_i^* = (v_{i1} v_{i2} \cdots v_{ip_i})$ where $1 \leq i \leq s$ be a permutation on 4n labels $v_{11}, v_{12}, \ldots, v_{1p_1}, v_{21}, v_{22}, \ldots, v_{2p_2}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_s}$. Let G be a graph constructed by Algorithm3.3 from the permutation σ^*. We note that G can also be constructed by Algorithm3.1 from the permutation σ^*. Hence by Theorem3.1(i) G is a s.c. graph. By the definition of the graph G it follows that σ^* is a star c.p. of G. For the label v_{i1} where

$1 \leq i \leq s-1$ define $v_{i2}, v_{i3}, \ldots, v_{i(\frac{p_i}{2}+1)}, v_{(i+1)1}, v_{(i+1)2}, \ldots, v_{(i+1)p_i}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_i}$ as the range of v_{i1}. For the label v_{s1} define $v_{s2}, v_{s3}, \ldots, v_{s(\frac{p_s}{2}+1)}$ as its range. We note that for any two vertices v_{il} and v_{jm} belonging to $Even(\sigma^*)$ where $1 \leq i < j \leq s$ the pair $[v_{il}, v_{jm}] = [\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})]$ for some odd integers k and r such that $1 \leq k \leq p_j - 1$ and v_{jr} is in the range of v_{i1}. By the definition of the graph G, $[v_{i1}, v_{jr}] \notin E(G)$ for each pair $[v_{i1}, v_{jr}]$ where $1 \leq i \leq s$ and v_{jr} is a vertex in the range of v_{i1} such that r is odd. Also by the definition of G the pair $[v_{i1}, v_{jr}] \notin E(G)$ implies $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ when k is odd such that $1 \leq k \leq p_j - 1$ for each pair $[v_{i1}, v_{jr}]$ where $1 \leq i \leq s$ and v_{jr} is a vertex in the range of v_{i1} such that r is odd. So $< Even(\sigma^*) >$ is a complete subgraph of G. By Theorem3.8 it follows that G is a chordal graph.

(ii) Let G be a s.c. chordal graph with p=4n where n is a positive integer. Let $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ where $\sigma_i^* = (v_{i1} v_{i2} \ldots v_{ip_i})$ for all $1 \leq i \leq s$ be a star c.p. of G. By Theorem3.8 $< Even(\sigma^*) >$ is a complete subgraph of G. So $Odd(\sigma^*)$ is a stable set of G since $Odd(\sigma^*) = \sigma^*(Even(\sigma^*))$. Then by Lemma2.2 it follows that G can be constructed by Algorithm3.3 from the permutation σ^*. □

We illustrate Algorithm3.3 by constructing a s.c. chordal graph with 12 vertices. Two positive integers $p_1 = 4$ and $p_2 = 8$ are chosen. We note that the positive integers p_1 and p_2 are such that $p_1 + p_2 = 12$, $p_1 \leq p_2$, $p_1 = 2^2$ and $p_2 = 2^3$. Let $\sigma^* = (v_{11}v_{12}v_{13}v_{14})(v_{21}v_{22}v_{23}v_{24}v_{25}v_{26}v_{27}v_{28})$ be a permutation on 12 labels v_{ij} where $i = 1, 2$ and $1 \leq j \leq p_i$. For the label v_{11} the labels $v_{12}, v_{13}, v_{21}, v_{22}, v_{23}$ and v_{24} constitute its range. For the label v_{21} the labels v_{22}, v_{23}, v_{24} and v_{25} constitute its range. We construct a graph G with 12 vertices by identifying the vertices of G with the labels v_{ij} where $i = 1, 2$ and $1 \leq j \leq p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_{i1}, v_{jr}]$ where $i = 1, 2$ and v_{jr} is a vertex in the range of v_{i1} the pair $[v_{i1}, v_{jr}] \notin E(G)$ when r is odd and it is arbitrarily decided whether $[v_{i1}, v_{jr}] \in E(G)$ or $[v_{i1}, v_{jr}] \notin E(G)$ when r is even. Let the vertex v_{11} be adjacent to the vertices v_{12} and v_{22} and non-adjacent to the vertex v_{24} of its range as shown Figure 3.5(a). Let the vertex v_{21} be adjacent to the vertex v_{24} and non-adjacent to the vertex v_{22} of its range as shown in Figure3.5(a).

(b) After completing (a) whether $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ or $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin E(G)$

for all $1 \leq k \leq p_j - 1$ is determined as follows for each pair $[v_{i1}, v_{jr}]$ where $i = 1, 2$ and v_{jr} is in the range of v_{i1}. The pair $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is even and $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is odd.

The labelled graph G constructed is shown in Figure 3.5(b). We note that σ^* is a star c.p. of the s.c. chordal graph G.

Remark: Algorithm3.3 may construct the same graph (with different labellings) repetetively. For example consider the graphs shown in Figure 3.6(b) and Figure 3.7(b) which are constructed by Algorithm3.3 with the adjacency schemes shown in Figure 3.6(a) and Figure 3.7(a) respectively. We note that these two graphs are isomorphic. So after constructing all s.c. chordal graphs with 4n vertices from Algorithm3.3 to compile a catalogue of s.c. chordal graphs with 4n vertices every pair of the constructed graphs should be tested for isomorphism.

The following algorithm constructs all s.c. chordal graphs with 4n+1 vertices.

Algorithm 3.4 : An algorithm to construct s.c. chordal graphs with 4n+1 vertices, n a positive integer

Choose positive integers $p_2, p_3, \ldots, p_s$ such that $\sum_{i=2}^{s} p_i = 4n$, $p_i \leq p_j$ for all $2 \leq i < j \leq s$ and $p_i = 2^{k_i}$ for some $k_i \geq 2$ where $2 \leq i \leq s$. Let $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ where $\sigma_1^* = (v_0)$ and $\sigma_i^* = (v_{i1} v_{i2} \ldots v_{ip_i})$ for all $2 \leq i \leq s$ be a permutation on 4n+1 labels $v_0, v_{21}, v_{22}, \ldots, v_{2p_2}, v_{31}, v_{32}, \ldots, v_{3p_3}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_s}$. For the label v_0 define $v_{21}, v_{31}, \ldots, v_{s1}$ as its range. For the label v_{i1} where $2 \leq i \leq s-1$ define $v_{i2}, v_{i3}, \ldots, v_{i(\frac{p_i}{2}+1)}, v_{(i+1)1}, v_{(i+1)2}, \ldots, v_{(i+1)p_i}, v_{(i+2)1}, v_{(i+2)2}, \ldots, v_{(i+2)p_i}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_i}$ as the range of v_{i1}. For the label v_{s1} define $v_{s2}, v_{s3}, \ldots, v_{s(\frac{p_s}{2}+1)}$ as its range. Construct a graph G with 4n+1 vertices by identifying the vertices of G with the labels v_0 and v_{ij} where $2 \leq i \leq s$, $1 \leq j \leq p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_0, v_{i1}]$ where v_{i1} is in the range of v_0 the pair $[v_0, v_{i1}] \notin E(G)$.

(b) After completing (a) whether the pairs $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \in E(G)$ or $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \notin E(G)$ for all $1 \leq k \leq p_i - 1$ is determined as follows for each pair $[v_0, v_{i1}]$ such that v_{i1} is in the range of v_0. The pair $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \notin E(G)$ when k is even and $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \in E(G)$ when k is odd.

(c) For each pair $[v_{i1}, v_{jr}]$ where $2 \leq i \leq s$ and v_{jr} is a vertex in the range of v_{i1} the pair $[v_{i1}, v_{jr}] \notin E(G)$ when r is odd and it is arbitrarily decided whether $[v_{i1}, v_{jr}] \in E(G)$ or $[v_{i1}, v_{jr}] \notin E(G)$ when r is even.

(d) After completing (c) whether the pairs $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ or $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin E(G)$ for all $1 \leq k \leq p_j - 1$ is determined as follows for each pair $[v_{i1}, v_{jr}]$ such that $2 \leq i \leq s$ and v_{jr} is in the range of v_{i1}. The pair $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is even and the pair $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is odd.

Let G be a s.c. graph with a c.p. σ. The following Theorem establishes a relation between the degree of a vertex and the degree of its image under σ.

Theorem 3.10 : *Let G be a s.c. graph with p vertices. Let $\sigma = \sigma_1\sigma_2 \cdots \sigma_s$ where $\sigma_i = (v_{i1}v_{i2} \cdots v_{ip_i})$ for all $1 \leq i \leq s$ be a c.p. of G. Then $deg_G(v_{ij}) + deg_G(\sigma(v_{ij})) = p - 1$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$.*

Proof: We note that $deg_G(v_{ij}) = deg_{\bar{G}}(\sigma(v_{ij}))$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$ since σ is a vertex isomorphism of G onto $\bar{G}$. Also $deg_{\bar{G}}(\sigma(v_{ij})) = p - 1 - deg_G(\sigma(v_{ij}))$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$. Hence $deg_G(v_{ij}) + deg_G(\sigma(v_{ij})) = p - 1$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$. □

Corollary 3.1 : *Let G be a s.c. chordal graph with p=4n+1. Let σ be a c.p. of G with v_0 as its fixed vertex. Then $deg_G(v_0) = 2n$.*

Proof: We note that $\sigma(v_0) = v_0$ since v_0 is the fixed vertex of σ. Hence from Theorem3.10 $deg_G(v_0) = 2n$. □

Let G be a s.c. chordal graph with 4n+1 vertices. Let σ be a c.p. of G with v_0 as its fixed vertex. The following Theorem gives the structure of the induced subgraph of G induced by the vertices adjacent to v_0.

Theorem 3.11 : *Let G be a s.c. chordal graph with p=4n+1 and a c.p. σ. Let v_0 be the fixed vertex of σ. Then $< Nhd_G(v_0) > \cong K_{2n}$.*

Proof: By Theorem2.20 $\omega(G) = 2n+1$. Let C be a maximum clique of G. Let V_C be the vertex set of C. We prove that $v_0 \in V_C$. Let $v_0 \notin V_C$. We note that C is a maximum clique of $G-v_0$. Then $\omega(G-v_0) = 2n+1$. By Corollary2.1 $G-v_0$ is a s.c. graph. So by Theorem2.11 $\omega(G-v_0) \leq 2n$, a contradiction. Hence $v_0 \in V_C$. By Corollary3.1 $deg_G(v_0) = 2n$. Then it follows that $< Nhd_G(v_0) > \cong K_{2n}$. □

Let G be a s.c. chordal graph with 4n+1 vertices. Let σ^* be a star c.p. of G with v_0 as its fixed vertex. The following Theorem states that the vertices adjacent to v_0 are precisely the vertices in $Even(\sigma^*)$.

Theorem 3.12 : *Let G be a s.c. chordal graph with p=4n+1 and a star c.p. σ^*. Let v_0 be the fixed vertex of σ^*. Then $Nhd_G(v_0) = Even(\sigma^*)$.*

Proof: Let $\sigma^* = \sigma_1^*\sigma_2^* \cdots \sigma_s^*$ where $\sigma_i^* = (v_{i1}v_{i2}\cdots v_{ip_i})$. By Corollary3.1 $|Nhd_G(v_0)| = 2n$. So to show that $Nhd_G(v_0) = Even(\sigma^*)$, we prove that $Nhd_G(v_0) \cap (Odd(\sigma^*) - \{v_0\}) = \emptyset$ for both the sets $Nhd(v_0)$ and $Even(\sigma^*)$ have 2n elements. Let a vertex $v_{ij} \in Nhd_G(v_0) \cap (Odd(\sigma^*) - \{v_0\})$. By Corollary2.2 $G - v_0$ is a s.c. graph with a star c.p. σ^*/σ_1^*. So by Theorem2.11 and Theorem3.11 $\omega(G - v_0) = 2n$. By Theorem2.14 $< Even(\sigma^*/\sigma_1^*) >$ is a complete subgraph of $G - v_0$ and hence a complete subgraph of G. It follows that $< Even(\sigma^*) >$ is a complete subgraph of G since $Even(\sigma^*) = Even(\sigma^*/\sigma_1^*)$. Since the induced subgraph $< Even(\sigma^*) >$ of G is a complete subgraph of G, it implies that $Odd(\sigma^*) - \{v_0\}$ is a stable set of G since $Odd(\sigma^*) - \{v_0\} = \sigma^*(Even(\sigma^*))$. The vertex v_{ij} belongs to $Odd(\sigma^*) - \{v_0\}$ implies that $\sigma^{*^2}(v_{ij})$ is a vertex distinct from v_{ij} such that $\sigma^{*^2}(v_{ij}) \in Odd(\sigma^*) - \{v_0\}$ since by Theorem2.1 the length of every permutation cycle of σ^* is atleast 4 except the cycle (v_0). We note that $[v_{ij}, v_0] \in E(G)$ since $v_{ij} \in Nhd_G(v_0)$. Also by Lemma2.2 $[v_{ij}, v_0] \in E(G)$ implies $[\sigma^{*^2}(v_{ij}), \sigma^{*^2}(v_0)] = [\sigma^{*^2}(v_{ij}), v_0] \in E(G)$. Then $\sigma^{*^2}(v_{ij}) \in Nhd_G(v_0)$. The pair $[v_{ij}, \sigma^{*^2}(v_{ij})] \notin E(G)$ since $v_{ij} \in Odd(\sigma^*)-\{v_0\}$, $\sigma^{*^2}(v_{ij}) \in Odd(\sigma^*)-\{v_0\}$ and $Odd(\sigma^*)-\{v_0\}$ is a stable set of G. Hence $Nhd_G(v_0)$ is not a complete subgraph of G, a contradiction to Theorem3.11. Therefore $Nhd_G(v_0) \cap (Odd(\sigma^*) - \{v_0\}) = \emptyset$. □

The following Theorem is a characterisation of a s.c. graph G with 4n+1 vertices to be a chordal graph in terms of its induced subgraph $< Even(\sigma^*) \cup \{v_0\} >$ where σ^* is a star c.p. of G with v_0 as its fixed vertex.

Theorem 3.13 : *Let G be a s.c. graph with 4n+1 vertices. Let σ^* be a star c.p. of G with v_0 as its fixed vertex. Then G is chordal if and only if $< Even(\sigma^*) \cup \{v_0\} >$ is a complete subgraph of G.*

Proof: Let G be a chordal graph. By Theorem3.11 and Theorem3.12 $< Even(\sigma^*) \cup \{v_0\} >$ is a complete subgraph of G.

Let $< Even(\sigma^*) \cup \{v_0\} >$ be a complete subgraph of G. Then $\omega(G) \geq 2n+1$ since $|Even(\sigma^*)| = 2n$. So by Theorem2.11 it follows that $\omega(G) = 2n+1$. Hence by Theorem2.20 G is a chordal graph. □

The following Theorem states that a graph G with 4n+1 vertices is a s.c. chordal graph if and only if it can be constructed by Algorithm3.4.

Theorem 3.14 : *(i) The graphs constructed by Algorithm3.4 are s.c. chordal graphs with 4n+1 vertices where n is a positive integer.*
(ii) Every s.c. chordal graph with 4n+1 vertices for all positive integers n can be constructed by Algorithm3.4.

Proof: (i) Let n be a positive integer. Let $p_2, p_3, \ldots, p_s$ be positive integers such that $\sum_{i=2}^{s} p_i = 4n$, $p_i \leq p_j$ for all $2 \leq i < j \leq s$ and $p_i = 2^{k_i}$ for some $k_i \geq 2$ where $2 \leq i \leq s$. Let $\sigma^* = \sigma_1^* \sigma_2^* \ldots \sigma_s^*$ where $\sigma_1^* = (v_0)$ and $\sigma_i^* = (v_{i1} v_{i2} \cdots v_{ip_i})$ where $2 \leq i \leq s$ be a permutation on 4n+1 labels $v_0, v_{21}, v_{22}, \ldots, v_{2p_2}, v_{31}, v_{32}, \ldots, v_{3p_3}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_s}$. Let G be a graph constructed by Algorithm3.4 from the permutation σ^*. We note that G can also be constructed by Algorithm3.2 from the permutation σ^*. Hence by Theorem3.2(i) G is a s.c. graph. By the definition of the graph G it follows that σ^* is a star c.p. of G. For the label v_{i1} where $2 \leq i \leq s-1$ define $v_{i2}, v_{i3}, \ldots, v_{i(\frac{p_i}{2}+1)}, v_{(i+1)1}, v_{(i+1)2}, \ldots, v_{(i+1)p_i}, \ldots, v_{s1}, v_{s2}, \ldots, v_{sp_i}$ as the range of v_{i1}. For the label v_{s1} define $v_{s2}, v_{s3}, \ldots, v_{s(\frac{p_s}{2}+1)}$ as its range. We note that for any two vertices v_{il} and v_{jm} belonging to $Even(\sigma^*)$ where $2 \leq i < j \leq s$ the pair $[v_{il}, v_{jm}] = [\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})]$ for some odd integers k and r such that $1 \leq k \leq p_j - 1$ and v_{jr} is in the range of v_{i1}. By the definition of the graph G, $[v_{i1}, v_{jr}] \notin E(G)$ for each pair $[v_{i1}, v_{jr}]$ where $2 \leq i \leq s$ and v_{jr} is a vertex in the range of v_{i1} such that r is odd. Also by the definition of G the pair $[v_{i1}, v_{jr}] \notin E(G)$ implies $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ when k is

odd such that $1 \leq k \leq p_j - 1$ for each pair $[v_{i1}, v_{jr}]$ where $2 \leq i \leq s$ and v_{jr} is a vertex in the range of v_{i1} such that r is odd. So $< Even(\sigma^*) >$ is a complete subgraph of G. By the definition of G, $[v_0, v_{i1}] \notin E(G)$ for all $2 \leq i \leq s$. Also for all $v_{ij} \in Even(\sigma^*)$ where $2 \leq i \leq s$ the pair $[v_0, v_{ij}] = [\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})]$ where k is some odd integer such that $1 \leq k \leq p_i - 1$. So $[v_0, v_{ij}] \in E(G)$ for all $v_{ij} \in Even(\sigma^*)$ since $[v_0, v_{i1}] \notin E(G)$ for all $2 \leq i \leq s$. Therefore $< \{v_0\} \cup Even(\sigma^*) >$ is a complete subgraph of G. By Theorem3.13 it follows that G is a chordal graph.

(ii) Let G be a s.c. chordal graph with p=4n+1 where n is a positive integer. Let $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ where $\sigma_1^* = (v_0)$ and $\sigma_i^* = (v_{i1} v_{i2} \ldots v_{ip_i})$ for all $2 \leq i \leq s$ be a star c.p. of G. By Theorem3.13 $< \{v_0\} \cup Even(\sigma^*) >$ is a complete subgraph of G. So $Odd(\sigma^*) \cup \{v_0\}$ is a stable set of G since $Odd(\sigma^*) \cup \{v_0\} = \sigma^*(Even(\sigma^*) \cup \{v_0\})$. Then by Lemma2.2 it follows that G can be constructed by Algorithm3.4 from the permutation σ^*. □

We illustrate Algorithm3.4 by constructing a s.c. chordal graph with 13 vertices. Two positive integers $p_2 = 4$ and $p_3 = 8$ are chosen. We note that $p_2 + p_3 = 12$, $p_2 \leq p_3$, $p_2 = 2^2$ and $p_3 = 2^3$. Let $\sigma^* = (v_0)(v_{21}v_{22}v_{23}v_{24})(v_{31}v_{32}v_{33}v_{34}v_{35}v_{36}v_{37}v_{38})$ be a permutation on 13 labels v_0 and v_{ij} where $i = 2, 3$, $1 \leq j \leq p_i$. For the label v_0 the labels v_{21} and v_{31} constitute its range. For the label v_{21} the labels $v_{22}, v_{23}, v_{31}, v_{32}, v_{33}$ and v_{34} constitute its range. For the label v_{31} the labels v_{32}, v_{33}, v_{34} and v_{35} constitute its range. We construct a graph G with 13 vertices by identifying the vertices of G with the labels v_0 and v_{ij} where $i = 2, 3$, $1 \leq j \leq p_i$ and the adjacencies of the vertices of G are determined as follows.

(a) For each pair $[v_0, v_{i1}]$ where v_{i1} is in the range of v_0 the pair $[v_0, v_{i1}] \notin E(G)$ as shown in Figure 3.8(a).

(b) After completing (a) whether the pair $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \in E(G)$ or $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \notin E(G)$ for all $1 \leq k \leq p_i - 1$ is determined as follows for each pair $[v_0, v_{i1}]$ such that v_{i1} is in the range of v_0. The pair $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \in E(G)$ if and only if $[v_0, v_{i1}] \in E(G)$ when k is even and $[\sigma^{*^k}(v_0), \sigma^{*^k}(v_{i1})] \notin E(G)$ if and only if $[v_0, v_{i1}] \in E(G)$ when k is odd.

(c) For each pair $[v_{i1}, v_{jr}]$ where $i = 2, 3$ and v_{jr} is a vertex in the range of v_{i1} the pair $[v_{i1}, v_{jr}] \notin E(G)$ when r is odd and it is arbitrarily decided whether $[v_{i1}, v_{jr}] \in E(G)$ or $[v_{i1}, v_{jr}] \notin E(G)$ when r is even. Let the vertex v_{21} be adjacent to the vertices v_{22} and v_{32} and non-adjacent to the vertex v_{34} of its range as shown in Figure 3.8(a). Let the vertex

v_{31} be adjacent to the vertex v_{34} and non-adjacent to the vertex v_{32} of its range as shown in Figure 3.8(a).

(d) After completing (c) whether $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ or $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin E(G)$ for all $1 \leq k \leq p_j - 1$ is determined as follows for each pair $[v_{i1}, v_{jr}]$ where $i = 2, 3$ and v_{jr} is in the range of v_{i1}. The pair $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \in E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is even and $[\sigma^{*^k}(v_{i1}), \sigma^{*^k}(v_{jr})] \notin E(G)$ if and only if $[v_{i1}, v_{jr}] \in E(G)$ when k is odd.

The labelled graph G constructed is shown in Figure 3.8(b). We note that σ^* is a star c.p. of the s.c. chordal graph G.

Remark: Algorithm3.4 may construct the same graph (with different labellings) many times. For example consider the graphs shown in Figure 3.9(b) and Figure 3.10(b) which are constructed by Algorithm3.4 with the adjacency schemes shown in Figure 3.9(a) and Figure 3.10(a) respectively. The two graphs constructed by Algorithm3.4 are isomorphic. To compile a catalogue of s.c. chordal graphs with 4n+1 vertices, every pair of the constructed graphs should be tested whether the graphs are isomorphic or not after constructing all s.c. chordal graphs with 4n+1 vertices from Algorithm3.4.

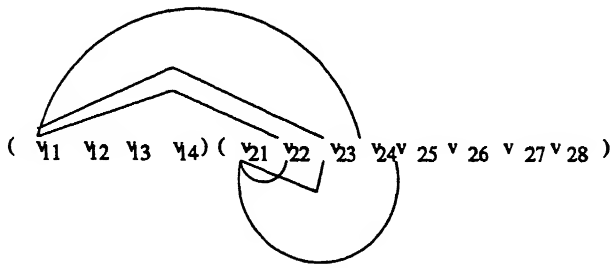

Figure 3.1a

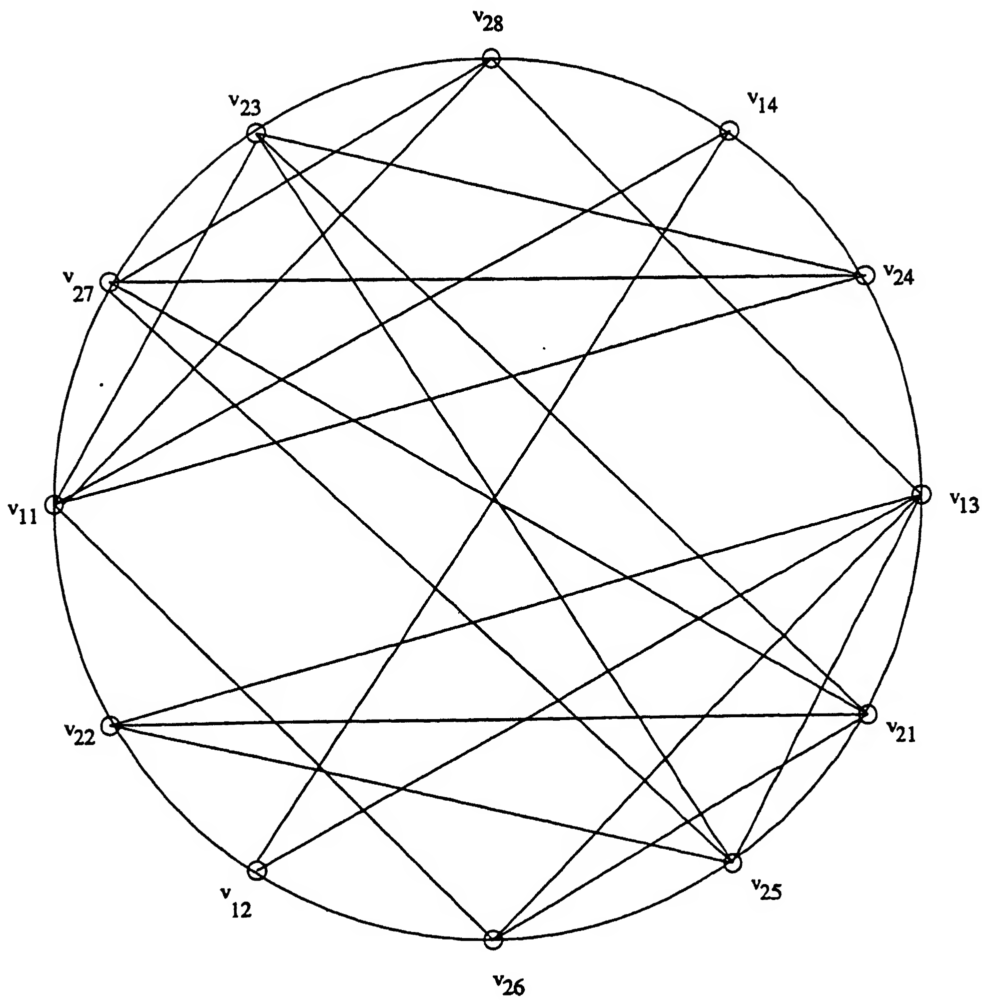

Figure 3.1b

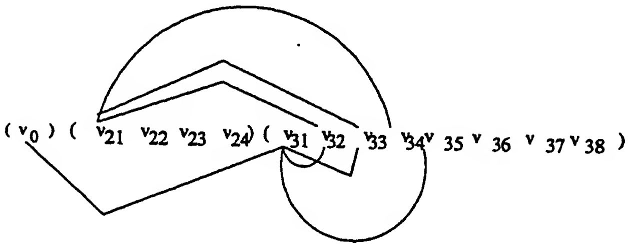

Figure 3.2a

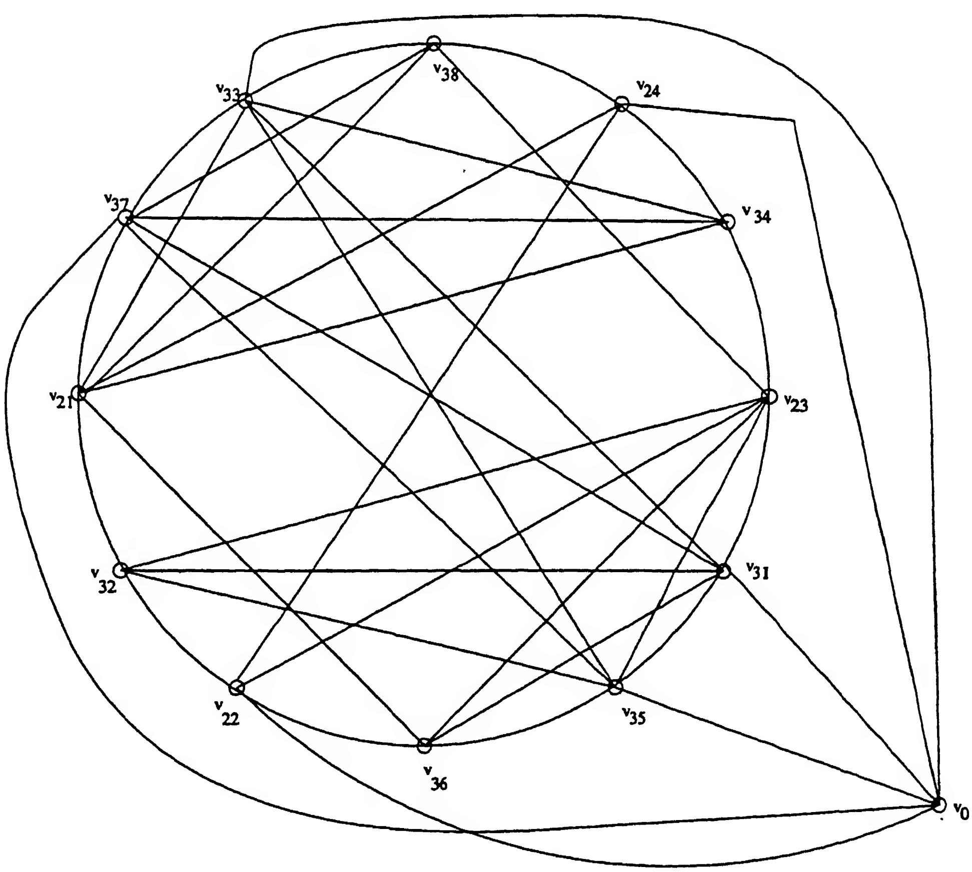

Figure 3.2b

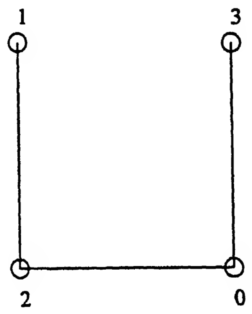

Figure 3.3a

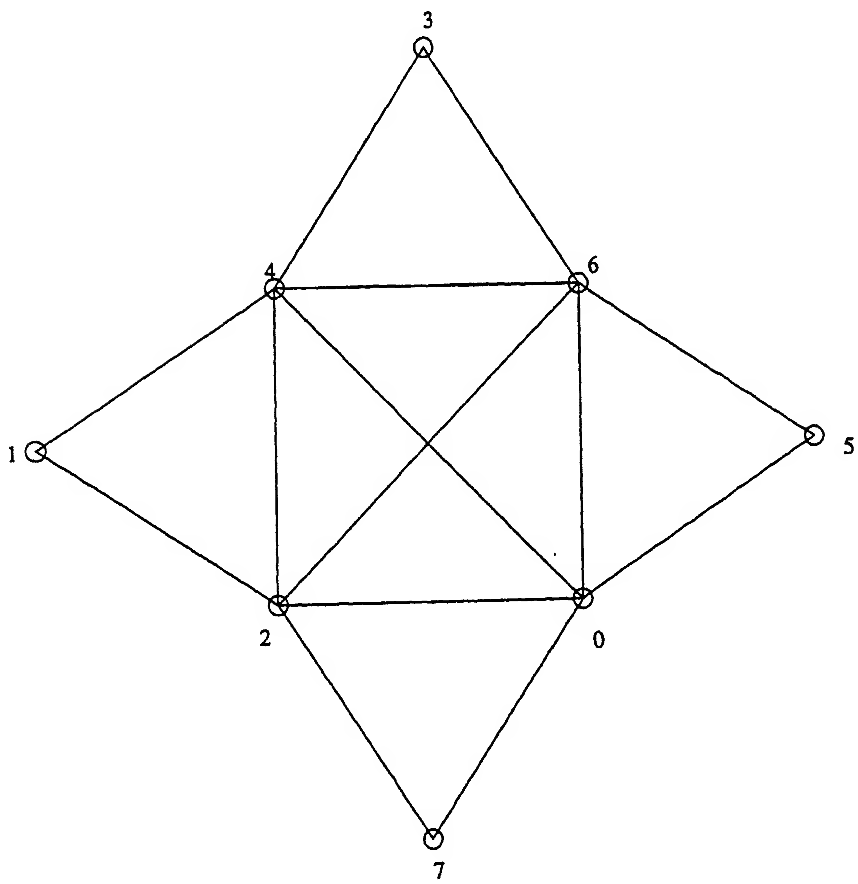

Figure 3.3b

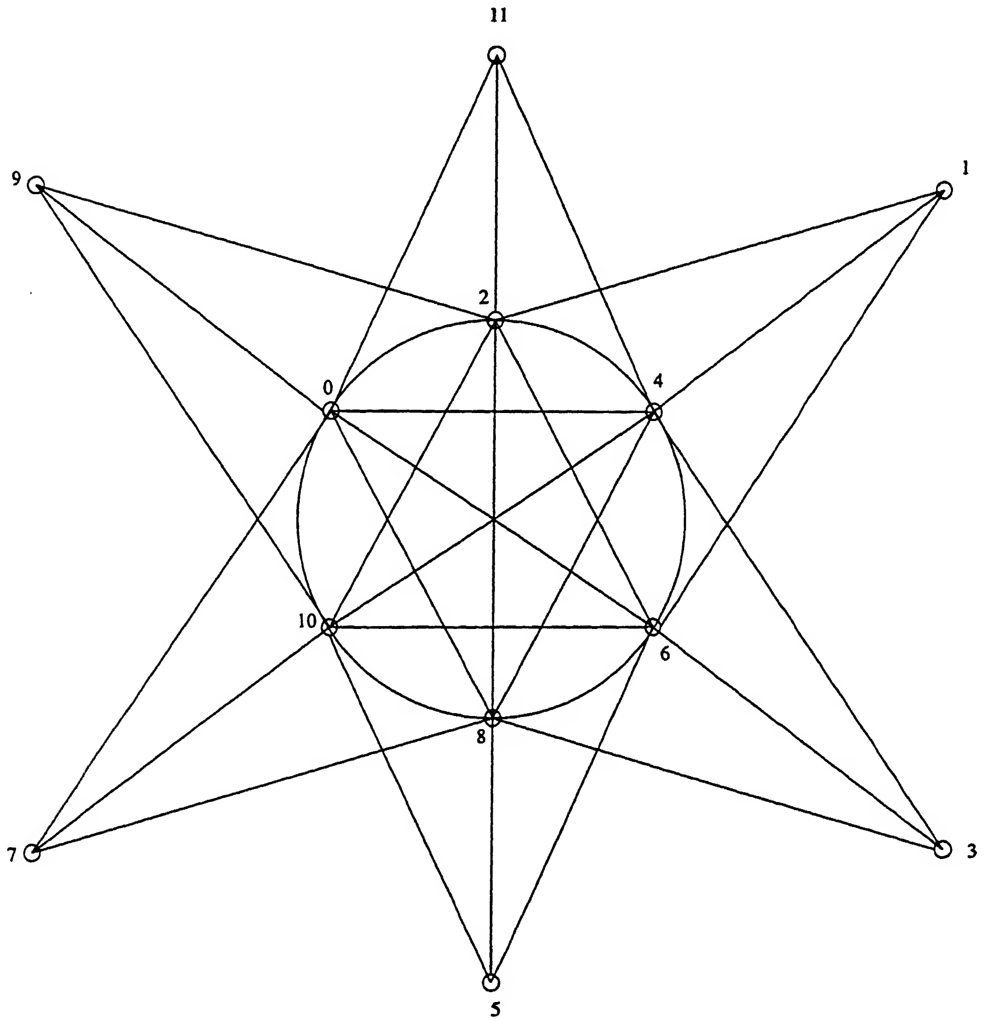

Figure 3.3c

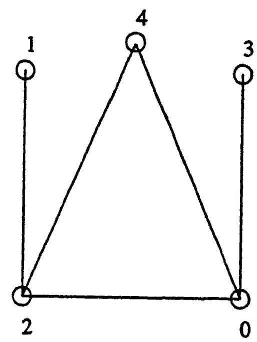

Figure 3.4a

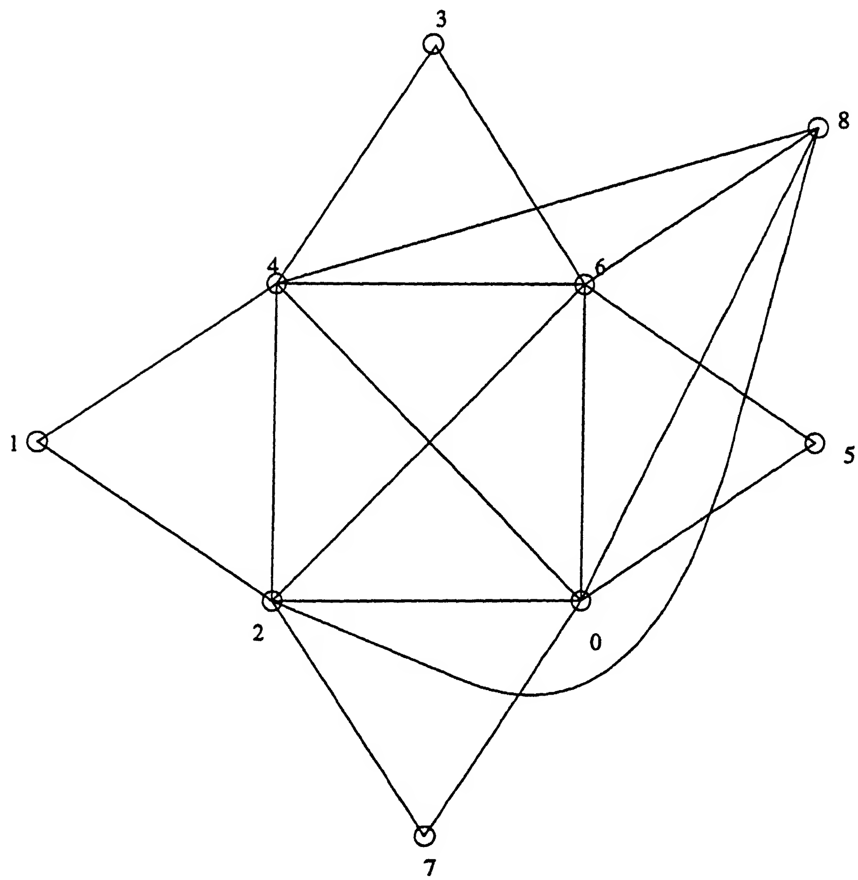

Figure 3.4b

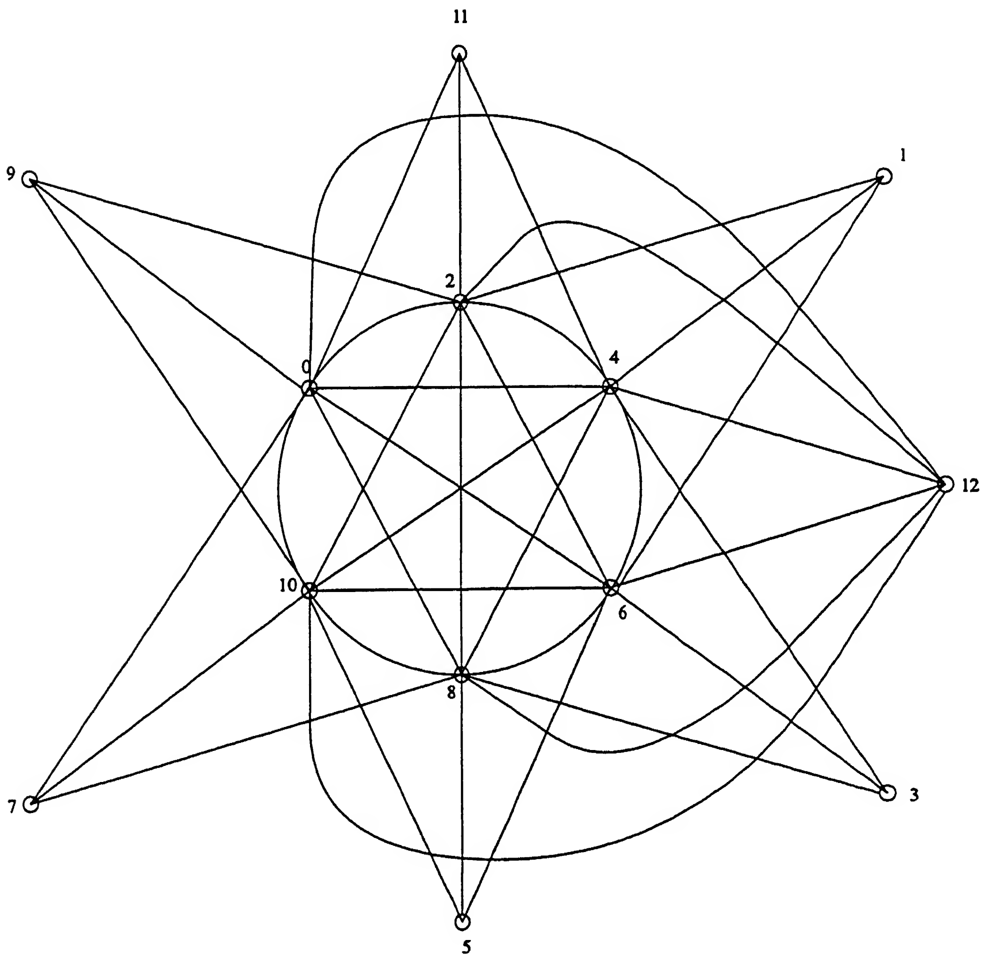

Figure 3.4c

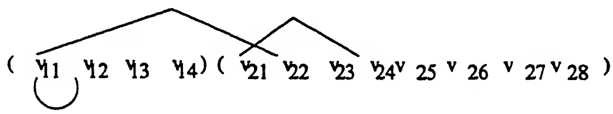

Figure 3.5a

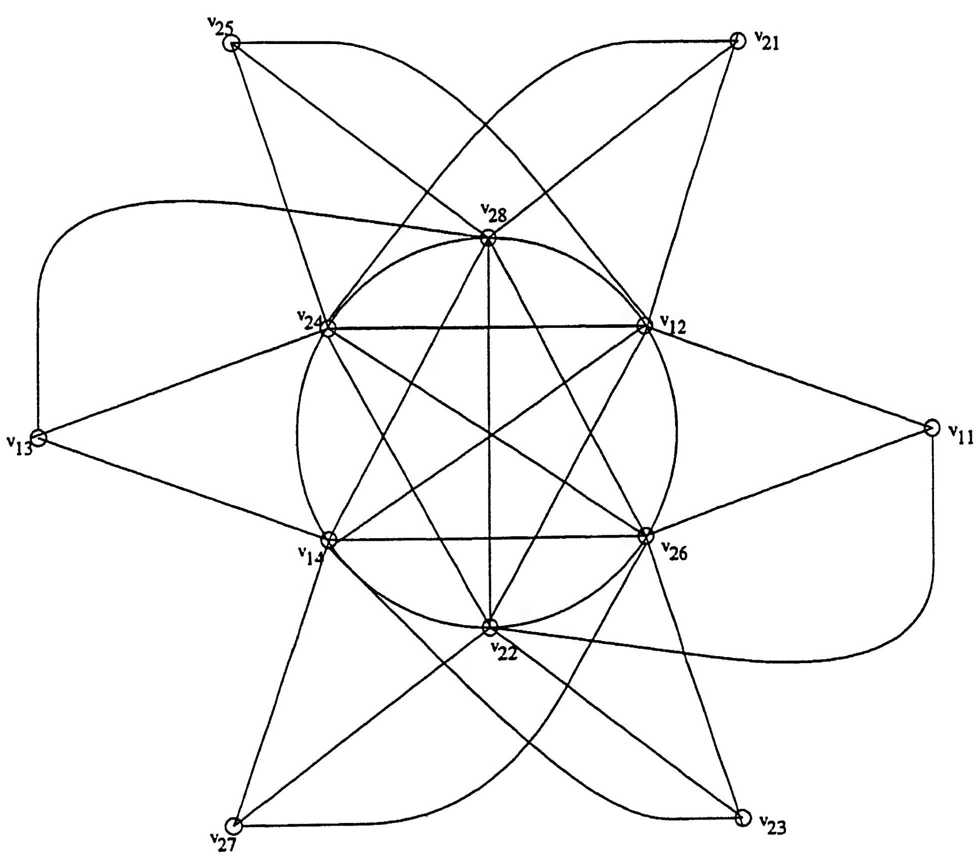

Figure 3.5b

$$(v_{11}\ v_{12}\ v_{13}\ v_{14})\ (v_{21}\ v_{22}\ v_{23}\ v_{24})$$

Figure 3.6a

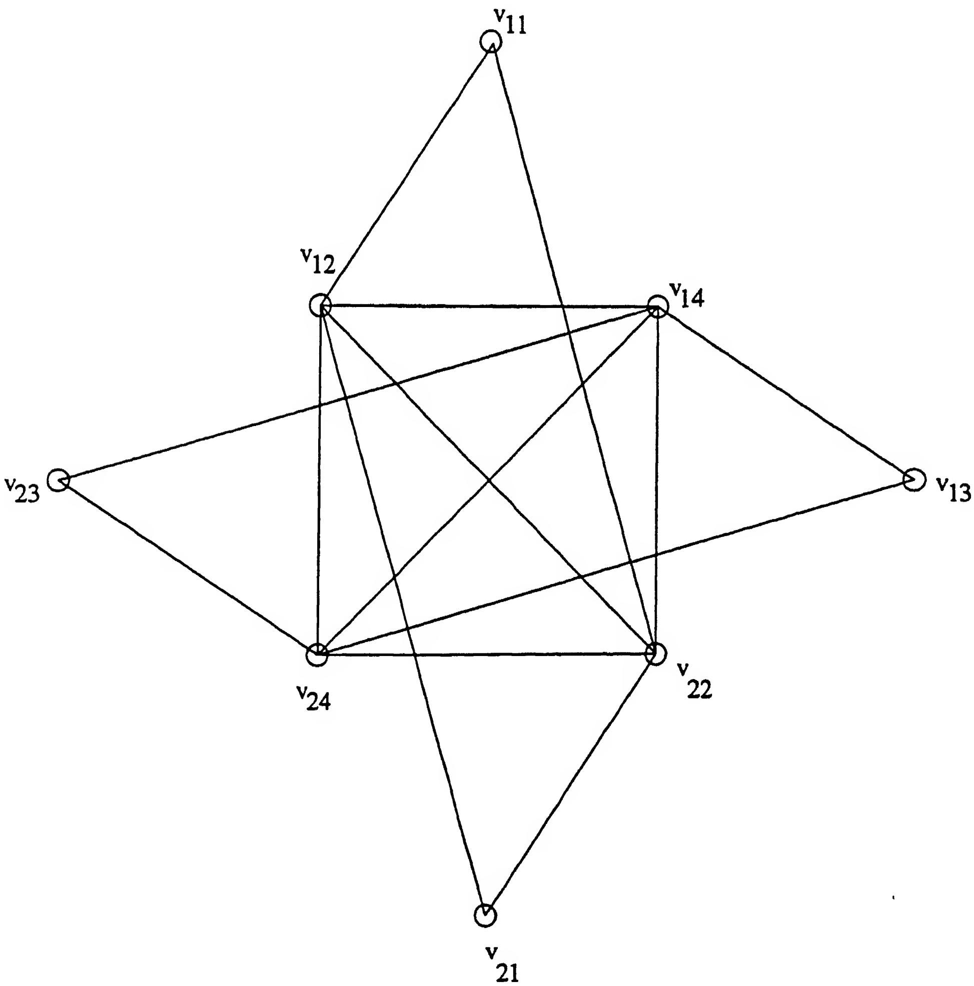

Figure 3.6b

$(v_{11}\ v_{12}\ v_{13}\ v_{14}\ v_{15}\ v_{16}\ v_{17}\ v_{18})$

Figure 3.7a

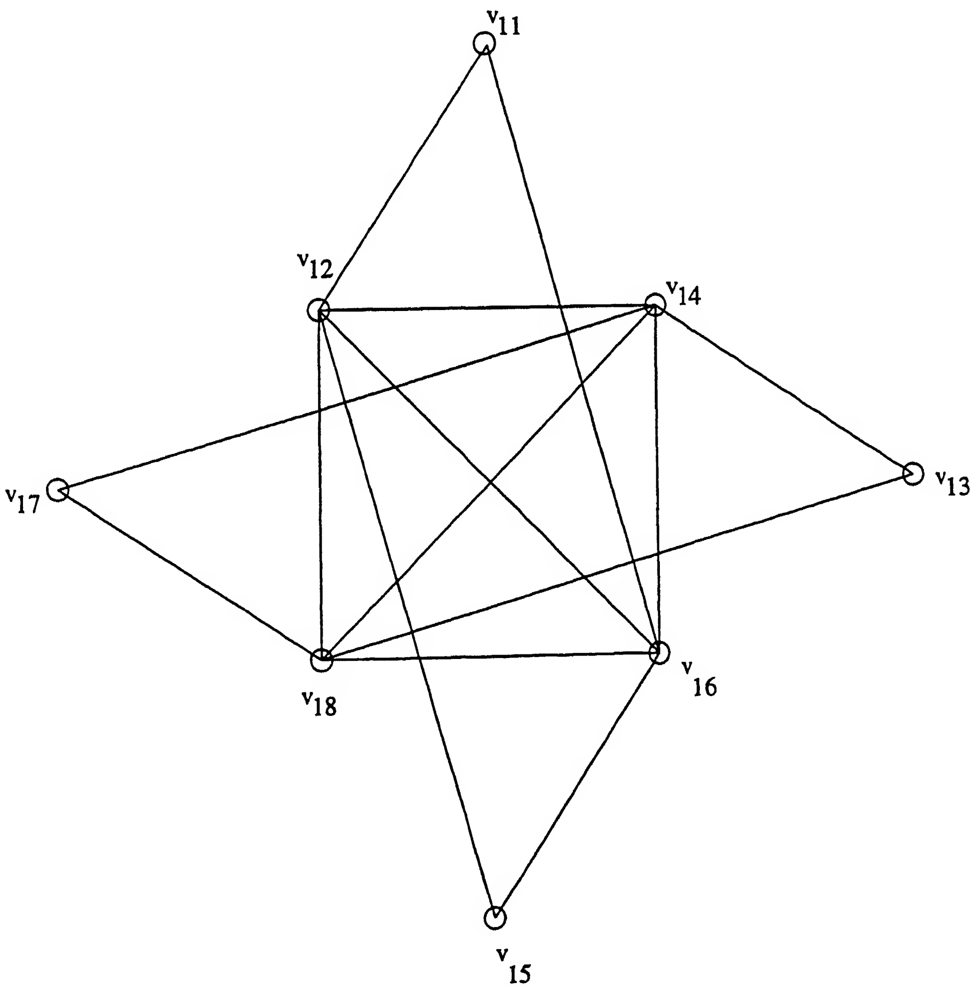

Figure 3.7b

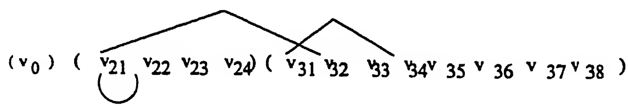

Figure 3.8a

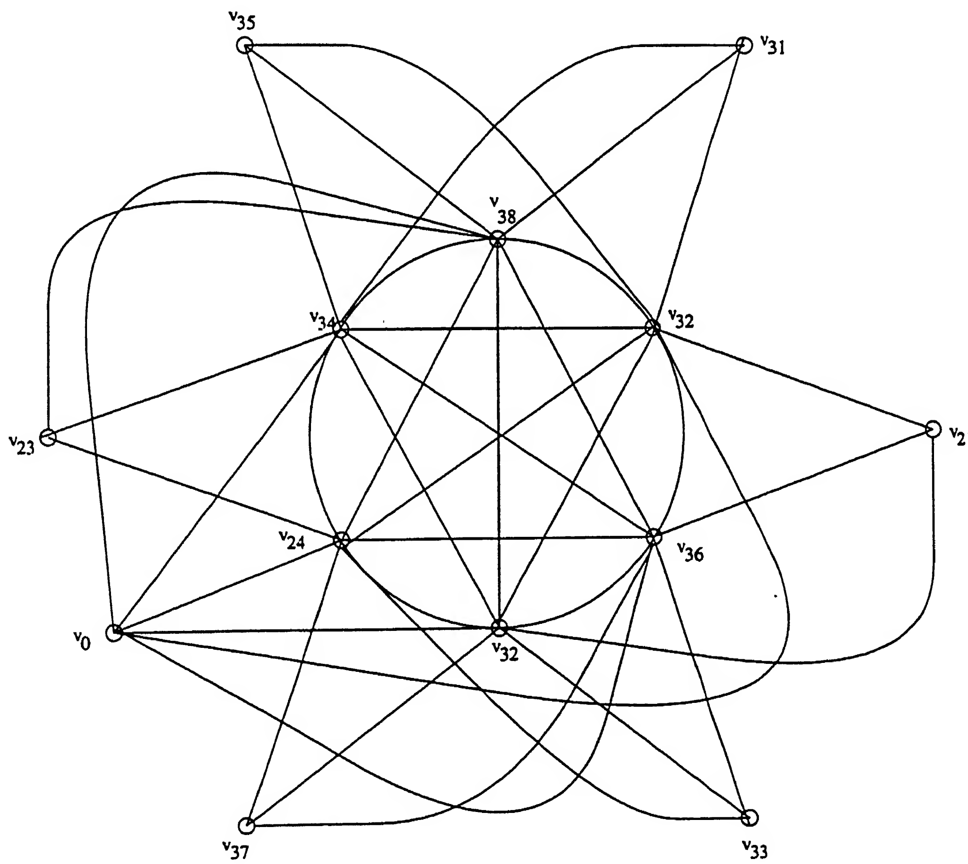

Figure 3.8b

$(v_0)(v_{21}\ v_{22}\ v_{23}\ v_{24})\ (v_{31}\ v_{32}\ v_{33}\ v_{34})$

Figure 3.9a

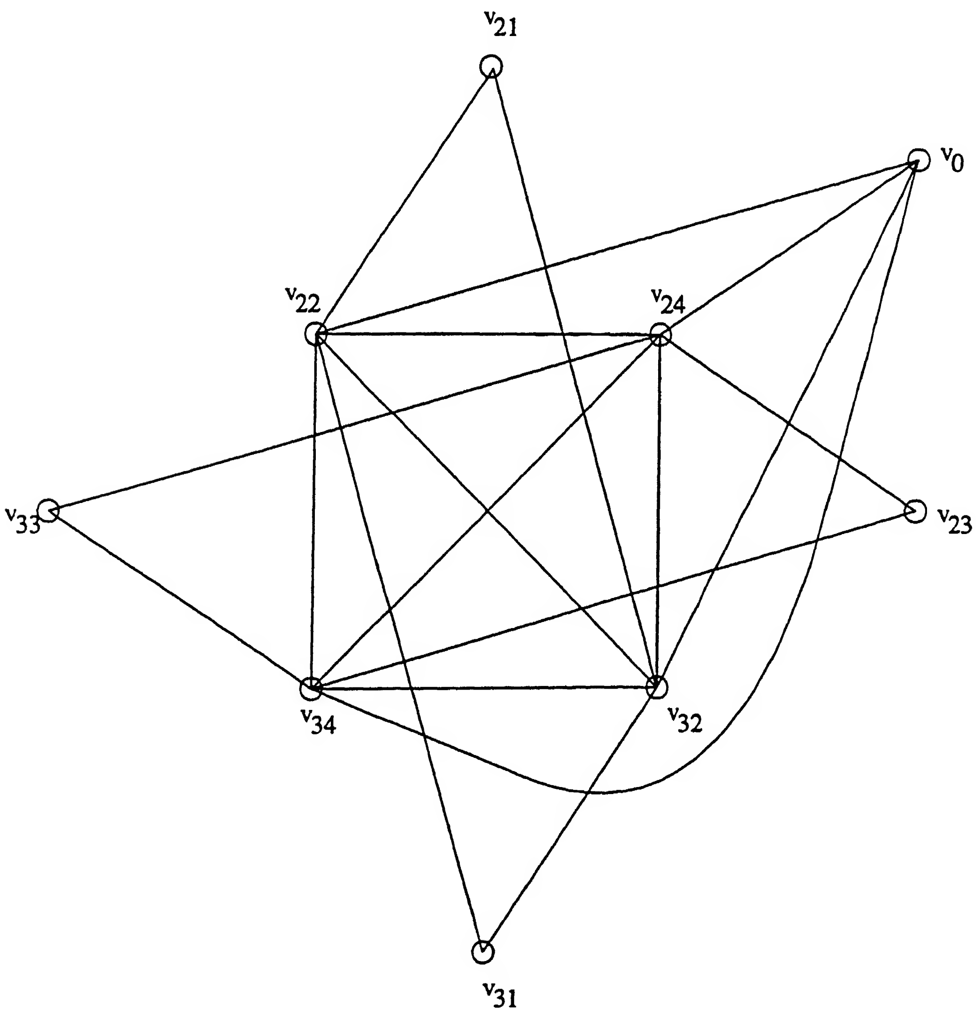

Figure 3.9b

$(v_0)(v_{21}\ v_{22}\ v_{23}\ v_{24}\ v_{25}\ v_{26}\ v_{27}\ v_{28})$

Figure 3.10a

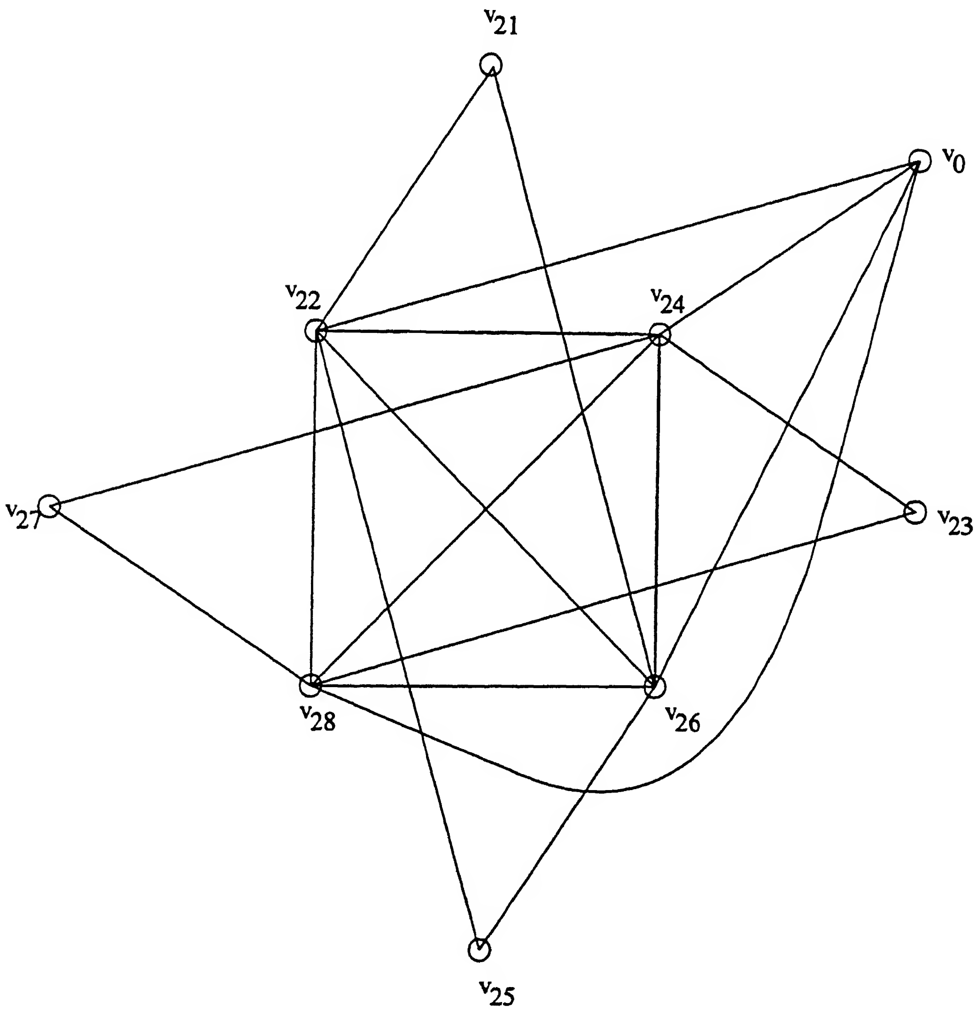

Figure 3.10b

Chapter 4

On the isomorphism and the catalogue compilation of self-complementary chordal graphs

4.1 Introduction

Colbourn et. al. [65] and [66] prove that the isomorphism of s.c. graphs and the isomorphism of regular s.c. graphs are polynomially equivalent to the isomorphism problem. In Section4.2 of this chapter we prove that the isomorphism of s.c chordal graphs, the isomorphism of s.c. chordal graphs with 4n vertices and the isomorphism of s.c. chordal graphs with 4n+1 vertices are polynomially equivalent. Catalogue of s.c. graphs with 8 vertices had been compiled by Alter [5], Faradzhev [78], Morris [143] and Venkatachalam [225]. Faradzhev [78] and Morris [143] also compiled the catalogue of s.c. graphs with 9 vertices. Faradzhev [78] and Kropar et. al. [127] obtained the catalogue of s.c. graphs with 12 vertices. Kropar and Read in [127] observed that obtaining catalogue of s.c. graphs with more than 12 vertices was quite difficult. In the case of s.c. chordal graphs too the compilation of catalogue seems to be quite difficult. This difficulty is due to its close relationship with the isomorphism of s.c. chordal graphs as observed in the previous chapter. However we could compile the catalogue of s.c. chordal graphs with atmost 13 vertices using the existing catalogue of s.c.

graphs. In Section 4.3 of this chapter we give linear time algorithms to decide whether a given s.c. graph with 4n or 4n+1 vertices is chordal or not. Using this algorithm, from the available catalogue of s.c. graphs with atmost 12 vertices we locate those graphs which are chordal and obtain the catalogue of s.c. chordal graphs with atmost 12 vertices. We also obtain the catalogue of s.c. chordal graphs with 13 vertices by giving a method for obtaining all non-isomorphic s.c. chordal graphs with 4n+1 vertices from the set of all non-isomorphic s.c. chordal graphs with 4n vertices.

4.2 Algorithmic complexity of the isomorphism of self-complementary chordal graphs

Lemma 4.1 : *Let G' and G'' be such that $G' \cong G''$. Let ψ be a vertex isomorphism of G' onto G''. Then $G' - v \cong G'' - \psi(v)$.*

Proof: Follows from the definition of ψ the vertex isomorphism of G' onto G''. □

For a s.c. graph G with 4n vertices and a star c.p. σ^* the following result gives an upper bound and a lower bound for the degrees of the vertices in $Odd(\sigma^*)$ and $Even(\sigma^*)$ respectively.

Theorem 4.1 : *Let G be a s.c. chordal graph with p=4n and a star c.p. σ^*. Then*

(i) $deg_G(v_{ij}) \leq 2n-1$ *for all* $v_{ij} \in Odd(\sigma^*)$

(ii) $deg_G(v_{ij}) \geq 2n$ *for all* $v_{ij} \in Even(\sigma^*)$.

Proof: (i) We note that $\sigma^*(Even(\sigma^*)) = Odd(\sigma^*)$. Also we note that $\omega(G) = 2n$ by Theorem2.20. Let $deg_G(v_{ij}) \geq 2n$ for some vertex $v_{ij} \in Odd(\sigma^*)$. By Theorem2.14 $< Even(\sigma^*) >$ is a maximum clique of G. Hence $Odd(\sigma^*)$ is a stable set of G. So the vertex v_{ij} can be adjacent with only even labelled vertices of σ^* in G. We note that v_{ij} is adjacent with all even labelled vertices of σ^* since $deg_G(v_{ij}) \geq 2n$, $|Even(\sigma^*)| = 2n$ and v_{ij} is adjacent with only even labelled vertices of σ^*. Hence $< \{v_{ij}\} \cup Even(\sigma^*) >$ is a complete subgraph of G, a contradiction. Therefore $deg_G(v_{ij}) \leq 2n-1$ for all $v_{ij} \in Odd(\sigma^*)$.

(ii) Let $deg_G(v_{ij}) \leq 2n-1$ for some vertex $v_{ij} \in Even(\sigma^*)$. By Theorem3.10 $deg_G(\sigma^*(v_{ij})) \geq 2n$, a contradiction to (i) of this Theorem since $\sigma^*(v_{ij}) \in Odd(\sigma^*)$. Therefore $deg_G(v_{ij}) \geq 2n$ for all $v_{ij} \in Even(\sigma^*)$. □

For a s.c. graph G with 4n+1 vertices and a star c.p. σ^* the following result gives an upper bound and a lower bound for the degrees of the vertices in $Odd(\sigma^*) - \{v_0\}$ and $Even(\sigma^*)$ respectively where v_0 is the fixed vertex of σ^*.

Theorem 4.2 : *Let G be a s.c. chordal graph with p=4n+1 and a star c.p. σ^*. Let v_0 be the fixed vertex of σ^*. Then*

(i) $deg_G(v_{ij}) \leq 2n-1$ *for all* $v_{ij} \in Odd(\sigma^*) - \{v_0\}$

(ii) $deg_G(v_{ij}) \geq 2n+1$ *for all* $v_{ij} \in Even(\sigma^*)$.

Proof: Let $G' = G - v_0$. By Corollary2.2 G' is a s.c. graph with a star c.p. σ^*/σ_1^*. So by Theorem4.1, $deg_{G'}(v_{ij}) \leq 2n-1$ for all $v_{ij} \in Odd(\sigma^*/\sigma_1^*)$ and $deg_{G'}(v_{ij}) \geq 2n$ for all $v_{ij} \in Even(\sigma^*/\sigma_1^*)$. Also $Odd(\sigma^*/\sigma_1^*) = Odd(\sigma^*) - \{v_0\}$ and $Even(\sigma^*/\sigma_1^*) = Even(\sigma^*)$. Hence $deg_G(v_{ij}) \leq 2n-1$ for all $v_{ij} \in Odd(\sigma^*) - \{v_0\}$ and $deg_G(v_{ij}) \geq 2n+1$ for all $v_{ij} \in Even(\sigma^*)$ since by Theorem3.12 $Nhd_G(v_0) = Even(\sigma^*)$. □

Let G be a s.c. chordal graph with 4n vertices. The graph $G_U \diamond u_0$ obtained from G is also a s.c. chordal graph.

Theorem 4.3 : *Let G be a s.c. chordal graph with p=4n. Let* $U = \{v \in V(G) : deg_G(v) \geq 2n\}$ *and u_0 be a vertex not belonging to V(G). Then $G_U \diamond u_0$ is a s.c. chordal graph.*

Proof: By Theorem 2.20 we note that $\omega(G) = 2n$. Let $\sigma^* = \sigma_1^*\sigma_2^* \cdots \sigma_s^*$ where $\sigma_i^* = (v_{i1}v_{i2}\cdots v_{ip_i})$ for $1 \leq i \leq s$ be a star c.p. of G. Let E' be the edge set of the graph $G_U \diamond u_0$. Let $\sigma^{*'} = (u_0)\sigma^*$. We prove that $\sigma^{*'}$ is a c.p. of $G_U \diamond u_0$. By the definition of the graph $G_U \diamond u_0$ the pair $[u_0, v_{ij}] \in E'$ if and only if $deg_G(v_{ij}) \geq 2n$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$. By Theorem3.10 $deg_G(v_{ij}) \geq 2n$ if and only if $deg_G(\sigma^*(v_{ij})) \leq 2n-1$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$. We note that $deg_G(\sigma^*(v_{ij})) \leq 2n-1$ if and only if $[u_0, \sigma^*(v_{ij})] \notin E'$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$ by the definition of the graph $G_U \diamond u_0$. Hence $[u_0, v_{ij}] \in E'$ if and

only if $[u_0, \sigma^*(v_{ij})] \notin E'$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$. Therefore $[u_0, v_{ij}] \in E'$ if and only if $[\sigma^{*'}(u_0), \sigma^{*'}(v_{ij})] \notin E'$, since $[\sigma^{*'}(u_0), \sigma^{*'}(v_{ij})] = [u_0, \sigma^{*'}(v_{ij})]$ for all $1 \leq i \leq s$ and $1 \leq j \leq p_i$. For any two distinct vertices v_{ij} and v_{lm} such that $1 \leq i \leq s$, $1 \leq j \leq p_i$, $1 \leq l \leq s$ and $1 \leq m \leq p_l$ the pair $[v_{ij}, v_{lm}] \in E'$ if and only if $[\sigma^{*'}(v_{ij}), \sigma^{*'}(v_{lm})] = [\sigma^*(v_{ij}), \sigma^*(v_{lm})] \notin E'$ since σ^* is a star c.p. of G. Therefore $\sigma^{*'}$ is a c.p. of $G_U \diamond u_0$. So $G_U \diamond u_0$ is a s.c. graph. By Theorem2.11 $\omega(G_U \diamond u_0) \leq 2n+1$. We prove that $\omega(G_U \diamond u_0) \geq 2n+1$. By Theorem4.1 $U = Even(\sigma^*)$. By Theorem2.14 $< Even(\sigma^*) >$ is a complete subgraph of G since $\omega(G) = 2n$. Hence $< \{u_0\} \cup Even(\sigma^*) >$ is a complete subgraph of $G_U \diamond u_0$. This leads to $\omega(G_U \diamond u_0) \geq 2n+1$. So $\omega(G_U \diamond u_0) = 2n+1$. By Theorem2.20 the Theorem follows. □

Let G be a s.c. chordal graph with 4n+1 vertices and a c.p. σ. The graph $G - v_0$ obtained from G is also a s.c. chordal graph.

Theorem 4.4 : *Let G be a s.c. chordal graph with p=4n+1 and a c.p. σ. Let v_0 be the fixed vertex of σ. Then $G - v_0$ is a s.c. chordal graph.*

Proof: By Corollary2.1 G is a s.c. graph. We prove that $G - v_0$ is chordal by showing $\omega(G - v_0) = 2n$. By Theorem2.11 $\omega(G - v_0) \leq 2n$. By Theorem3.11 $< Nhd(v_0) > \cong K_{2n}$. Hence $\omega(G - v_0) \geq 2n$. Then $\omega(G - v_0) = 2n$. The result follows from Theorem2.20. □

Lemma 4.2 : *Let G be a s.c. chordal graph with p=4n+1. Let σ^* be a star c.p. of G with v_0 as its fixed vertex. Let $U = \{v \in V(G - v_0) \ : \ deg_{G-v_0}(v) \geq 2n\}$ and u_0 be a vertex not belonging to $V(G - v_0)$. Then $(G - v_0)_U \diamond u_0 \cong G$.*

Proof: Let $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ where $\sigma_1 = (v_0)$ and $\sigma_i = (v_{i1} v_{i2} \cdots v_{ip_i})$ for $2 \leq i \leq s$. By Theorem4.4 $G - v_0$ is a s.c. chordal graph. We note that σ^*/σ_1^* is a star c.p. of $G - v_0$. So by Theorem4.1 $U = Even(\sigma^*/\sigma_1^*)$. By Theorem3.12 $Nhd_G(v_0) = Even(\sigma^*)$. Hence $Nhd_G(v_0) = U$ since $Even(\sigma^*) = Even(\sigma^*/\sigma_1^*)$. Therefore by the definition of the graph $(G - v_0)_U \diamond u_0$ it follows that $(G - v_0)_U \diamond u_0 \cong G$. □

Lemma 4.3 : *Let G be a s.c. chordal graph with p=4n and a star c.p. σ^*. Let $U = \{v \in V(G) \ : \ deg_G(v) \geq 2n\}$ and u_0 be a vertex not belonging to $V(G)$. Let $\sigma^{*'}$ be a star c.p. of $G_U \diamond u_0$ with v_0' as its fixed vertex. Then $(G_U \diamond u_0) - v_0' \cong G$.*

Proof: By Theorem4.1 $U = Even(\sigma^*)$. So $Nhd_{G_U \diamond u_0}(u_0) = Even(\sigma^*)$. Therefore $|Nhd_{G_U \diamond u_0}(u_0)| = 2n$ since $|Even(\sigma^*)| = 2n$. Then $deg_{G_U \diamond u_0}(u_0) = 2n$. Hence by Theorem4.2 $u_0 = v_0'$. Therefore $(G_U \diamond u_0) - v_0' \cong G$. □

The following result states that the verification of whether two given s.c. chordal graphs with 4n vertices are isomorphic or not could be equivalently posed as a problem of verifying whether two given s.c. chordal graphs with 4n+1 vertices are isomorphic or not.

Theorem 4.5 : *Let G' and G'' be s.c. chordal graphs with 4n vertices. Let $U' = \{v \in V(G') \ : \ deg_{G'}(v) \geq 2n\}$ and $U'' = \{v \in V(G'') \ : \ deg_{G''}(v) \geq 2n\}$. Let u_0' and u_0'' be vertices not belonging to $V(G')$ and $V(G'')$ respectively. Then*

(i) *$G'_{U'} \diamond u_0'$ and $G''_{U''} \diamond u_0''$ are s.c. chordal graphs with 4n+1 vertices.*

(ii) *$G' \cong G''$ if and only if $G'_{U'} \diamond u_0' \cong G''_{U''} \diamond u_0''$.*

Proof: (i) Follows from Theorem 4.3.

(ii) Let $G' \cong G''$. Let ψ be a vertex isomorphism of G' onto G''. We note that $\psi(U') = U''$. We define a 1-1 correspondence $\tilde{\psi}$ from $V(G'_{U'} \diamond u_0')$ onto $V(G''_{U''} \diamond u_0'')$ as follows.

$$\tilde{\psi}(v) = \left[\begin{array}{ll} \psi(v) & if\ v \in V(G') \\ u_0'' & if\ v = u_0' \end{array} \right.$$

We prove that $\tilde{\psi}$ is a vertex isomorphism of $G'_{U'} \diamond u_0'$ onto $G''_{U''} \diamond u_0''$. Let $E' = E(G'_{U'} \diamond u_0')$ and $E'' = E(G''_{U''} \diamond u_0'')$. Let v be a vertex of $G'_{U'} \diamond u_0'$ distinct from u_0' . By the definition of the graph $G'_{U'} \diamond u_0'$ we note that $[u_0', v] \in E'$ if and only if $v \in U'$. By the definition of the graph $G''_{U''} \diamond u_0''$ the pair $[u_0'', \psi(v)] \in E''$ if and only if $\psi(v) \in U''$. Hence $[u_0', v] \in E'$ if and only if $[\tilde{\psi}(u_0'), \tilde{\psi}(v)] = [u_0'', \psi(v)] \in E''$ since $\psi(U') = U''$. Let u and v be two vertices of $G'_{U'} \diamond u_0'$ distinct from u_0' . We note that both u and v are vertices of G'. Hence $[u, v] \in E'$ if and only if $[\tilde{\psi}(u), \tilde{\psi}(v)] = [\psi(u), \psi(v)] \in E''$ since ψ is a vertex isomorphism of G' onto G''. Therefore $\tilde{\psi}$ is a vertex isomophism of $G'_{U'} \diamond u_0'$ onto $G''_{U''} \diamond u_0''$. Hence $G'_{U'} \diamond u_0' \cong G''_{U''} \diamond u_0''$.

Let $G'_{U'} \diamond u_0' \cong G''_{U''} \diamond u_0''$. Let ψ be a vertex isomorphism of $G'_{U'} \diamond u_0'$ onto $G''_{U''} \diamond u_0''$. By (i) of this Theorem $G'_{U'} \diamond u_0'$ and $G''_{U''} \diamond u_0''$ are s.c. chordal graphs with 4n+1 vertices. Let $\sigma^{*'}$ and $\sigma^{*''}$ be star c.p.'s of $G'_{U'} \diamond u_0'$ and $G''_{U''} \diamond u_0''$ respectively. Let v_0' and v_0'' be the fixed vertices of $\sigma^{*'}$ and $\sigma^{*''}$ respectively. By Corollary3.1 $deg_{G'_{U'} \diamond u_0'}(v_0') = 2n$. Then

$deg_{G''_{U''} \diamond u''_0}(\psi(v'_0)) = deg_{G'_{U'} \diamond u'_0}(v'_0) = 2n$. Hence by Theorem4.2 $\psi(v'_0) = v''_0$. By Lemma4.1 $G'_{U'} \diamond u'_0 - v'_0 \cong G''_{U''} \diamond u''_0 - \psi(v'_0)$. So $(G'_{U'} \diamond u'_0) - v'_0 \cong (G''_{U''} \diamond u''_0) - v''_0$. By Lemma4.3 $(G'_{U'} \diamond u'_0) - v'_0 \cong G'$ and $(G''_{U''} \diamond u''_0) - v''_0 \cong G''$. This leads to $G' \cong G''$. □

The following result states that the verification of whether two given s.c. chordal graphs with 4n+1 vertices are isomorphic or not could be equivalently posed as a problem of verifying whether two given s.c. chordal graphs with 4n vertices are isomorphic or not.

Theorem 4.6 : *Let G' and G'' be s.c. chordal graphs with 4n+1 vertices. Let $\sigma^{*'}$ and $\sigma^{*''}$ be star c.p. of G' and G'' respectively. Let v'_0 and v''_0 be the fixed vertices of $\sigma^{*'}$ and $\sigma^{*''}$ respectively. Then*

(i) *$deg_{G'}(v'_0) = 2n$ and $deg_{G'}(v) \neq 2n$ for all vertices v of G' distinct from v'_0*

(ii) *$deg_{G''}(v''_0) = 2n$ and $deg_{G''}(v) \neq 2n$ for all vertices v of G'' distinct from v''_0*

(iii) *$G' - v'_0$ and $G'' - v''_0$ are s.c. chordal graphs with $4n$ vertices*

(iv) *$G' \cong G''$ if and only if $G' - v'_0 \cong G'' - v''_0$.*

Proof:(i) and (ii) follow from Corollary3.1 and Theorem4.2.
(iii) Follows from Theorem 4.4.
(iv) Let $U' = \{v \in V(G' - v'_0) \ : \ deg_{G'-v'_0}(v) \geq 2n\}$ and $U'' = \{v \in V(G'' - v''_0) \ : \ deg_{G''-v''_0}(v) \geq 2n\}$. Let u'_0 and u''_0 be vertices not belonging to $V(G' - v'_0)$ and $V(G'' - v''_0)$ respectively. By Theorem4.5(ii) $G' - v'_0 \cong G'' - v''_0$ if and only if $(G' - v'_0)_{U'} \diamond u'_0 \cong (G'' - v''_0)_{U''} \diamond u''_0$. By Lemma4.2 $(G' - v'_0)_{U'} \diamond u'_0 \cong G'$ and $(G'' - v''_0)_{U''} \diamond u''_0 \cong G''$. Hence $G' \cong G''$ if and only if $G' - v'_0 \cong G'' - v''_0$. □

The following algorithm constructs a s.c. chordal graph with 4n+1 vertices from a s.c. chordal graph with 4n vertices.

Algorithm 4.1 : **Given a s.c. chordal graph with 4n vertices this algorithm constructs a s.c. chordal graph with 4n+1 vertices**

Input : *A s.c. chordal graph with $4n$ vertices.*

Step 1 : *Locate all the vertices v such that $deg_G(v) \geq 2n$. Call the set of all such vertices as U.*

Output : *The graph $G_U \diamond u_0$ where u_0 is a vertex distinct from the vertices of G.*

The following result justifies the claim of Algorithm4.1.

Theorem 4.7 : *Let G be a s.c. chordal graph with 4n vertices. The output graph $G_U \diamond u_0$ obtained by Algorithm4.1 with G as its input graph is a s.c. chordal graph with 4n+1 vertices.*

Proof: Follows from Theorem4.3. □

The complexity of Algorithm4.1 is given by the following result.

Theorem 4.8 : *Algorithm4.1 is computable in linear time.*

Proof: Let a s.c. chordal graph G with 4n vertices be an input to Algorithm4.1. We note that Step1 of Algorithm4.1 takes O(n) time for the input G since it has to examine the degrees of the 4n vertices of G to locate all the vertices of degree atleast 2n. So Algorithm4.1 is computable in linear time. □

To illustrate Algorithm4.1 we consider the s.c. chordal graph G with 8 vertices shown in Figure 4.1(a). Let G be the input to Algorithm4.1. Step 1 of Algorithm4.1 locates all the vertices v of G such that $deg_G(v) \geq 4$. We note $U = \{2, 4, 6, 8\}$. The output $G_U \diamond u_0$ is shown in Figure 4.1(b).

The following algorithm constructs a s.c. chordal graph with 4n vertices from a s.c. chordal graph with 4n+1 vertices.

Algorithm 4.2 :Given a s.c. chordal graph with 4n+1 vertices this algorithm constructs a s.c. chordal graph with 4n vertices.

Input : *A s.c. chordal graph G with 4n+1 vertices.*

Step 1 : *Locate the vertex v of G such that $deg_G(v) = 2n$. Call that vertex as v_0.*

Output : *The graph $G - v_0$.*

The following result justifies the claim of Algorithm4.2.

Theorem 4.9 : *Let G be s.c. chordal graph with 4n+1 vertices. The output graph $G - v_0$ obtained by Algorithm4.2 with G as its input graph is a s.c. chordal graph with 4n vertices.*

Proof: Follows from Corollary3.1, Theorem4.2 and Theorem4.4. □

The complexity of Algorithm4.2 is given by the following result.

Theorem 4.10 : *Algorithm4.2 is computable in linear time.*

Proof: Let a s.c. chordal graph G with 4n+1 vertices be an input to Algorithm4.2. We note that Step1 of Algorithm4.2 takes O(n) time for the input G since it has to examine the degrees of atmost 4n+1 vertices of G to locate the vertex of degree 2n in G. Hence Algorithm4.2 is computable in linear time. □

To illustrate Algorithm4.2 we consider the s.c. chordal graph G with 9 vertices shown in Figure 4.2(a). Let G be the input to Algorithm4.2. Step 1 of Algorithm4.2 locates the vertex 9 which is the vertex of degree 4 in G. The output graph $G - v_0$ is shown in Figure4.2(b) where $v_0 = 9$.

Isomorphism of s.c. chordal graphs is algorithmically as hard as the isomorphism of some of its subclasses.

Theorem 4.11 : *The isomorphism of the following classes of graphs are polynomially equivalent.*

(i) *S.c. chordal graphs.*

(ii) *S.c. chordal graphs with 4n vertices where n is a positive integer.*

(iii) *S.c. chordal graphs with 4n+1 vertices where n is a positive integer.*

Proof: We prove that the isomorphism of (ii) is polynomially equivalent to the isomorphism of (iii).

Let G' and G'' be two s.c. chordal graphs with 4n vertices. Let $U' = \{v \in V(G') : deg_{G'}(v) \geq 2n\}$ and $U'' = \{v \in V(G'') : deg_{G''}(v) \geq 2n\}$. Let u_0' and u_0'' be vertices not belonging to $V(G')$ and $V(G'')$ respectively. Construct the graphs $G'_{U'} \diamond u_0'$ and $G''_{U''} \diamond u_0''$ by using Algorithm4.1. By Theorem4.7 the graphs $G'_{U'} \diamond u_0'$ and $G''_{U''} \diamond u_0''$ are s.c. chordal graphs with 4n+1 vertices. By Theorem4.5 G' is isomorphic to G'' if and only if $G'_{U'} \diamond u_0'$ is isomorphic to $G''_{U''} \diamond u_0''$. Hence the isomorphism of (ii) is polynomially reducible to the isomorphism of (iii) since by Theorem4.8 Algorithm4.1 is polynomially computable.

Let G' and G'' be two s.c. chordal graphs with 4n+1 vertices. Construct the graphs $G' - v_0'$ and $G'' - v_0''$ by using Algorithm4.2 where v_0' and v_0'' are the vertices of G' and G'' such that $deg_{G'}(v_0') = 2n$ and $deg_{G''}(v_0'') = 2n$. By Theorem4.9 the graphs $G' - v_0'$ and $G'' - v_0''$ are s.c. chordal graphs with 4n vertices. By Theorem4.6 G' is isomorphic to G'' if and only if $G' - v_0'$ is isomorphic to $G'' - v_0''$. Hence the isomorphism of (iii) is polynomially reducible to the isomorphism of (ii) since by Theorem4.10 Algorithm4.2 is polynomially computable.

Hence the isomorphism of (ii) is polynomially equivalent to the isomorphism of (iii).

Therefore the Theorem follows from the fact that every s.c. chordal graph has 4n or 4n+1 vertices. □

4.3 On the catalogue compilation of self-complementary chordal graphs

The following algorithm recognises whether a given s.c. graph with 4n vertices is chordal or not.

Algorithm 4.3 : Given a s.c. graph G with 4n vertices this algorithm decides whether G is chordal or not.

Input : *A s.c. graph G with 4n vertices.*

Step 1 : *Compute the degrees of the vertices of G.*

Step 2 : *Arrange the degrees of the vertices of G in non-increasing order. Let $d_1 \geq d_2 \geq \cdots \geq d_{4n}$ be the sequence of the degrees of the vertices of G arranged in non-increasing order.*

Step 3 : *Compute the sum $\sum_{i=1}^{2n} d_i$.*

Output : *Whether the graph G is chordal or not is decided as follows. The graph G is chordal if $\sum_{i=1}^{2n} d_i = 6n^2 - 2n$ else it is not chordal.*

The following result justifies the claim of Algorithm4.3.

Theorem 4.12 : *Let G be a s.c. graph with $4n$ vertices where n is a positive integer. Then G is a chordal graph if and only if it is decided as a chordal graph by Algorithm4.3.*

Proof: Follows from Theorem2.34 (i). □

The complexity of Algorithm4.3 is given by the following result.

Theorem 4.13 : *Let G be a s.c. graph with $4n$ vertices where n is a positive integer. Algorithm4.3 with G as its input graph decides whether G is chordal or not in $O(n)$ time.*

Proof: Step1 of Algorithm4.3 takes O(n) time since all the 4n vertices of G should be examined to compute the degrees of the vertices of G. The degrees of the vertices of G are integers and take values between 0 and $4n-1$. So it takes O(n) time for sorting the degrees of the 4n vertices of G [1]. Step3 of Algorithm4.3 takes O(1) time. Therefore the result. □

To illustrate Algorithm4.3 we consider the graphs G' and G'' shown in Figure 4.3(a) and Figure 4.3(b) respectively.

Let G' be the graph shown in Figure 4.3(a). The graph G' is a s.c. graph since (1 2 3 4 5 6 7 8 9 10 11 12) is a c.p. of G'. Let G' be the input to Algorithm4.3. Step 1 of Algorithm4.3 yields $deg_{G'}(2) = deg_{G'}(4) = deg_{G'}(6) = deg_{G'}(8) = deg_{G'}(10) = deg_{G'}(12) = 8$ and $deg_{G'}(1) = deg_{G'}(3) = deg_{G'}(5) = deg_{G'}(7) = deg_{G'}(9) = deg_{G'}(11) = 3$. Step 2 of Algorithm4.3 arranges the degrees of the vertices of G' in non-increasing order $d_1 \geq d_2 \geq \cdots \geq d_{12}$ where $d_i = 8$ for $1 \leq i \leq 6$ and $d_i = 3$ for $7 \leq i \leq 12$. By Step 3 of Algorithm4.3 $\sum_{i=1}^{6} d_i = 48$. Since $6n^2 - 2n = 48 = \sum_{i=1}^{6} d_i$ Algorithm4.3 decides the graph G' is a chordal graph.

Let G'' be the graph shown in Figure 4.3(b). The graph G'' is s.c. since (1 2 3 4 5 6 7 8) is a c.p. of G''. Let G'' be the input to Algorithm4.3. Step 1 of Algorithm4.3 yields $deg_{G''}(2) = deg_{G''}(4) = deg_{G''}(6) = deg_{G''}(8) = 4$ and $deg_{G''}(1) = deg_{G''}(3) = deg_{G''}(5) = deg_{G''}(7) = 3$. Step 2 of Algorithm4.3 arranges the degrees of the vertices of G'' in non-increasing order $d_1 \geq d_2 \geq \cdots \geq d_8$ where $d_i = 4$ for $1 \leq i \leq 4$ and $d_i = 3$ for $5 \leq i \leq 8$. By Step 3 of Algorithm4.3 $\sum_{i=1}^{4} d_i = 16$. Since $6n^2 - 2n = 20 \neq \sum_{i=1}^{4} d_i$ Algorithm4.3 decides the graph G'' is not a chordal graph.

The following algorithm recognises whether a given s.c. graph with 4n+1 vertices is chordal or not.

Algorithm 4.4 : Given a s.c. graph G with 4n+1 vertices this algorithm decides whether G is chordal or not.

Input : *A s.c. graph G with 4n+1 vertices.*

Step 1 : *Compute the degrees of the vertices of G.*

Step 2 : *Arrange the degrees of the vertices of G in non-increasing order. Let $d_1 \geq d_2 \geq \cdots \geq d_{4n+1}$ be the sequence of the degrees of the vertices of G arranged in non-increasing order.*

Step 3 : *Compute the sum $\sum_{i=1}^{2n} d_i$.*

Output : *Whether the graph G is chordal or not is decided as follows. The graph G is chordal if $\sum_{i=1}^{2n} d_i = 6n^2$ else it is not chordal.*

The following result justifies the claim of Algorithm4.4.

Theorem 4.14 : *Let G be a s.c. graph with 4n+1 vertices where n is a positive integer. Then G is a chordal graph if and only if it is decided as a chordal graph by Algorithm4.4.*

Proof: Follows from Theorem2.34 (ii). □

The complexity of Algorithm4.4 is given by the following result.

Theorem 4.15 : *Let G be a s.c. graph with 4n+1 vertices where n is a positive integer. Algorithm4.4 with G as its input graph decides whether G is chordal or not in O(n) time.*

Proof: Step1 of Algorithm4.4 takes O(n) time since all the 4n+1 vertices of G should be examined to compute the degrees of the vertices of G. The degrees of the vertices of G are integers and take values between 0 and 4n. So it takes O(n) time for sorting the degrees of the 4n+1 vertices of G [1]. Step3 of Algorithm4.4 takes O(1) time. Therefore the result. □

To illustrate Algorithm4.4 we consider the graphs G' and G'' shown in Figure 4.4(a) and Figure 4.4(b) respectively.

Let G' be the graph shown in Figure 4.4(a). The graph G' is s.c. since (1 2 3 4 5 6 7 8 9 10 11 12)(13) is a c.p. of G'. Let G' be the input to Algorithm4.4. Step 1 of Algorithm4.4 yields $deg_{G'}(2) = deg_{G'}(4) = deg_{G'}(6) = deg_{G'}(8) = deg_{G'}(10) = deg_{G'}(12) = 9$ and $deg_{G'}(1) = deg_{G'}(3) = deg_{G'}(5) = deg_{G'}(7) = deg_{G'}(9) = deg_{G'}(11) = 3$ and $deg_{G'}(13) = 6$. Step 2 of Algorithm4.4 arranges the degrees of the vertices of G' in non-increasing order $d_1 \geq d_2 \geq \cdots \geq d_{13}$ where $d_i = 9$ for $1 \leq i \leq 6$, $d_7 = 6$ and $d_i = 3$ for $8 \leq i \leq 13$. By Step 3 of Algorithm4.4 $\sum_{i=1}^{6} d_i = 54$. Since $6n^2 = 54 = \sum_{i=1}^{6} d_i$ Algorithm4.4 decides the graph G' is a chordal graph.

Let G'' be the graph shown in Figure 4.4(b). The graph G'' is s.c. since (1 2 3 4 5 6 7 8)(9) is a c.p. of G''. Let G'' be the input to Algorithm4.4. Step 1 of Algorithm4.4 yields $deg_{G''}(2) = deg_{G''}(4) = deg_{G''}(6) = deg_{G''}(8) = 5$, $deg_{G''}(1) = deg_{G''}(3) = deg_{G''}(5) = deg_{G''}(7) = 3$ and $deg_{G''} = 4$. Step 2 of Algorithm4.4 arranges the degrees of the vertices of G'' in non-increasing order $d_1 \geq d_2 \geq \cdots \geq d_9$ where $d_i = 4$ for $1 \leq i \leq 4$, $d_5 = 4$ and $d_i = 3$ for $6 \leq i \leq 9$. By Step 3 of Algorithm4.4 $\sum_{i=1}^{4} d_i = 16$. Since $6n^2 = 24 \neq \sum_{i=1}^{4} d_i$ Algorithm4.4 decides the graph G'' is not a chordal graph.

Remark: A graph can be recognised to be chordal or not in linear time using the algorithms given in [131], [184] and [186]. However recognising whether a chordal graph is self-complementary or not appears to be difficult.

Using Algorithm4.3 and Algorithm4.4 we compile the catalogue of s.c. chordal graphs with atmost 12 vertices from the available catalogue of s.c. graphs with atmost 12 vertices.

There is only one non-isomorphic s.c. chordal graph with 4 vertices. This graph is isomorphic to the graph with the following 4×4 matrix as its adjacency matrix.

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

There is only one non-isomorphic s.c. chordal graph with 5 vertices. This graph is isomorphic to the graph with the following 5×5 matrix as its adjacency matrix.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

There are 3 non-isomorphic s.c. chordal graphs with 8 vertices. They are isomorphic to the graphs with the following 8×8 matrices as its adjacency matrices.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

There are 3 non-isomorphic s.c. chordal graphs with 9 vertices. They are isomorphic to the graphs with the following 9×9 matrices as its adjacency matrices.

$$\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0
\end{pmatrix}$$

There are 16 non-isomorphic s.c. chordal graphs with 12 vertices. They are isomorphic to the graphs with the following 12×12 matrices as its adjacency matrices.

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&1&0&1&1\\
0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&1&1&1&1&1\\
0&0&0&0&0&0&1&0&1&1&1&1\\
0&0&0&0&0&1&0&1&1&1&1&1\\
0&0&0&0&1&0&1&0&1&1&1&1\\
0&0&1&0&1&1&1&1&0&1&1&1\\
0&0&0&1&1&1&1&1&1&0&1&1\\
0&1&1&1&1&1&1&1&1&1&0&1\\
1&0&1&1&1&1&1&1&1&1&1&0
\end{pmatrix}.$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&1&0&0&1&1&1\\
0&0&0&0&0&0&1&1&0&0&1&1\\
0&0&0&0&0&0&0&1&1&0&1&1\\
0&0&0&0&0&0&0&0&1&1&1&1\\
0&0&1&1&0&0&0&1&1&1&1&1\\
0&0&0&1&1&0&1&0&1&1&1&1\\
0&0&0&0&1&1&1&1&0&1&1&1\\
0&0&1&0&0&1&1&1&1&0&1&1\\
0&1&1&1&1&1&1&1&1&1&0&1\\
1&0&1&1&1&1&1&1&1&1&1&0
\end{pmatrix}.$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&1&0&1&0&1&1\\
0&0&0&0&0&0&0&1&0&1&1&1\\
0&0&0&0&0&0&1&0&1&0&1&1\\
0&0&0&0&0&0&0&1&0&1&1&1\\
0&0&1&0&1&0&0&1&1&1&1&1\\
0&0&0&1&0&1&1&0&1&1&1&1\\
0&0&1&0&1&0&1&1&0&1&1&1\\
0&0&0&1&0&1&1&1&1&0&1&1\\
0&1&1&1&1&1&1&1&1&1&0&1\\
1&0&1&1&1&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&1&0&0&1\\
0&0&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&1&0&1&0\\
0&0&0&0&0&0&0&1&1&1&1&1\\
0&0&0&0&0&0&1&0&1&1&1&1\\
0&0&0&0&0&1&0&1&1&1&1&1\\
0&0&0&0&1&0&1&0&1&1&1&1\\
1&0&0&1&1&1&1&1&0&1&1&1\\
0&1&1&0&1&1&1&1&1&0&1&1\\
0&1&0&1&1&1&1&1&1&1&0&1\\
1&0&1&0&1&1&1&1&1&1&1&0
\end{pmatrix}$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}
$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}
$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&0&0&1&0&0&1\\
0&0&0&0&0&0&0&1&1&0&0&1\\
0&0&0&0&0&0&1&0&0&1&1&0\\
0&0&0&0&0&0&0&1&1&0&1&1\\
0&0&0&0&0&0&1&0&0&1&1&1\\
0&0&0&1&0&1&0&1&1&1&1&1\\
0&0&1&0&1&0&1&0&1&1&1&1\\
0&1&1&0&1&0&1&1&0&1&1&1\\
1&0&0&1&0&1&1&1&1&0&1&1\\
1&0&0&1&1&1&1&1&1&1&0&1\\
0&1&1&0&1&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&1&0&0&0&1\\
0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&1&0&1&1\\
0&0&0&0&0&0&1&0&1&1&0&1\\
0&0&0&0&0&0&0&1&1&1&1&0\\
0&1&0&0&1&0&0&1&1&1&1&1\\
1&0&0&0&0&1&1&0&1&1&1&1\\
0&0&0&1&1&1&1&1&0&1&1&1\\
0&0&1&0&1&1&1&1&1&0&1&1\\
0&1&1&1&0&1&1&1&1&1&0&1\\
1&0&1&1&1&0&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&1&0&1&0\\
0&0&0&0&0&0&0&1&0&1&0&1\\
0&0&0&0&0&0&1&0&1&0&1&0\\
0&0&0&0&0&0&1&0&0&1&1&1\\
0&0&0&0&0&0&0&1&1&0&1&1\\
0&0&0&1&1&0&0&1&1&1&1&1\\
0&0&1&0&0&1&1&0&1&1&1&1\\
0&1&0&1&0&1&1&1&0&1&1&1\\
1&0&1&0&1&0&1&1&1&0&1&1\\
0&1&0&1&1&1&1&1&1&1&0&1\\
1&0&1&0&1&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&1&1&0&1&0&0\\
0&0&0&0&0&0&0&1&1&0&1&0\\
0&0&0&0&0&0&0&0&1&1&0&1\\
0&0&0&0&0&0&1&0&0&1&1&0\\
0&0&0&0&0&0&0&1&0&0&1&1\\
0&0&0&0&0&0&1&0&1&0&0&1\\
1&0&0&1&0&1&0&1&1&1&1&1\\
1&1&0&0&1&0&1&0&1&1&1&1\\
0&1&1&0&0&1&1&1&0&1&1&1\\
1&0&1&1&0&0&1&1&1&0&1&1\\
0&1&0&1&1&0&1&1&1&1&0&1\\
0&0&1&0&1&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&1&1&1&0&0\\
0&0&0&0&0&0&0&0&1&1&1&0\\
0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&1&0&0&0&1&1\\
0&0&0&0&0&0&1&1&0&0&0&1\\
1&0&0&0&1&1&0&1&1&1&1&1\\
1&1&0&0&0&1&1&0&1&1&1&1\\
1&1&1&0&0&0&1&1&0&1&1&1\\
0&1&1&1&0&0&1&1&1&0&1&1\\
0&0&1&1&1&0&1&1&1&1&0&1\\
0&0&0&1&1&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&1&0&0&1&0&1\\
0&0&0&0&0&0&0&1&1&0&1&0\\
0&0&0&0&0&0&1&0&0&1&0&1\\
0&0&0&0&0&0&0&1&1&0&1&0\\
0&0&0&0&0&0&0&1&1&0&1&0\\
0&0&0&0&0&0&1&0&0&1&0&1\\
1&0&1&0&0&1&0&1&1&1&1&1\\
0&1&0&1&1&0&1&0&1&1&1&1\\
0&1&0&1&1&0&1&1&0&1&1&1\\
1&0&1&0&0&1&1&1&1&0&1&1\\
0&1&0&1&1&0&1&1&1&1&0&1\\
1&0&1&0&0&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&1&0&1&0&1\\
0&0&0&0&0&0&1&0&1&0&1&0\\
0&0&0&0&0&0&0&1&0&1&0&1\\
0&0&0&0&0&0&1&0&1&0&1&0\\
0&0&0&0&0&0&1&0&0&1&0&1\\
0&0&0&0&0&0&0&1&1&0&1&0\\
0&1&0&1&1&0&0&1&1&1&1&1\\
1&0&1&0&0&1&1&0&1&1&1&1\\
0&1&0&1&0&1&1&1&0&1&1&1\\
1&0&1&0&1&0&1&1&1&0&1&1\\
0&1&0&1&0&1&1&1&1&1&0&1\\
1&0&1&0&1&0&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&1&0&1&0&1\\
0&0&0&0&0&0&1&0&1&0&1&0\\
0&0&0&0&0&0&1&0&0&1&0&1\\
0&0&0&0&0&0&1&1&0&0&1&0\\
0&0&0&0&0&0&0&1&1&0&0&1\\
0&0&0&0&0&0&0&0&1&1&1&0\\
0&1&1&1&0&0&0&1&1&1&1&1\\
1&0&0&1&1&0&1&0&1&1&1&1\\
0&1&0&0&1&1&1&1&0&1&1&1\\
1&0&1&0&0&1&1&1&1&0&1&1\\
0&1&0&1&0&1&1&1&1&1&0&1\\
1&0&1&0&1&0&1&1&1&1&1&0
\end{pmatrix}$$

Remark : We note that the catalogue of s.c. chordal graphs with p vertices can be compiled from the catalogue of s.c. graphs with p vertices for $p > 12$ by following the same method adopted for compiling the catalogue of s.c. chordal graphs with p vertices for $p \leq 12$. By Theorem4.13 and Theorem4.15 deciding whether a s.c. graph with 4n or 4n+1 vertices is chordal or not takes O(n) time. Scanning the entire list of non-isomorphic s.c. graphs with 4n or 4n+1 vertices takes $O(s_{4n})$ time or $O(s_{4n+1})$ time respectively. So the running time of the algorithm which locates the non-isomorphic s.c. chordal graphs with p vertices from the list of all non-isomorphic s.c. graphs with p vertices takes $O(ns_{4n})$ time or $O(ns_{4n+1})$ time accordingly as p=4n or p=4n+1 respectively.

The following result gives a method for constructing all s.c. chordal graphs with 4n+1 vertices from the set of all s.c. chordal graphs with 4n vertices.

Theorem 4.16 : *Let $\mathcal{G}$ be the set of all non-isomorphic s.c. chordal graphs with 4n vertices. Let $\mathcal{G}^* = \{G_U \diamond u_0 : G \in \mathcal{G}, u_0 \notin V(G) \text{ and } U = \{v \in V(G) : deg_G(v) \geq 2n\}\}$.*
Then $\mathcal{G}^*$ is the set of all non-isomorphic s.c. chordal graphs with 4n+1 vertices.

Proof: Let G' and G'' be two s.c. chordal graphs with 4n vertices. Let u_0' and u_0'' be such that $u_0' \notin V(G')$ and $u_0'' \notin V(G'')$. Let $U' = \{v \in V(G') : deg_{G'}(v) \geq 2n\}$ and $U'' = \{v \in V(G'') : deg_{G''}(v) \geq 2n\}$. By Theorem4.5 $G' \cong G''$ if and only if $G'_{U'} \diamond u_0' \cong G''_{U''} \diamond u_0''$. Hence the Theorem. □

Using Theorem4.16 we obtain the catalogue of s.c. chordal graphs with 13 vertices from the set of all non-isomorphic s.c. chordal graphs with 12 vertices.

There are 16 non-isomorphic s.c. chordal graphs with 13 vertices. They are isomorphic to the graphs with the following 13×13 matrices as its adjacency matrices.

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&1&1&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0\\
0&0&0&0&0&0&0&1&1&1&1&1&0\\
0&0&0&0&0&0&1&0&1&1&1&1&0\\
0&0&0&0&0&1&0&1&1&1&1&1&1\\
0&0&0&0&1&0&1&0&1&1&1&1&1\\
0&0&1&0&1&1&1&1&0&1&1&1&1\\
0&0&0&1&1&1&1&1&1&0&1&1&1\\
0&1&1&1&1&1&1&1&1&1&0&1&1\\
1&0&1&1&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&1&0&0&1&1&1&0\\
0&0&0&0&0&0&1&1&0&0&1&1&0\\
0&0&0&0&0&0&0&1&1&0&1&1&0\\
0&0&0&0&0&0&0&0&1&1&1&1&0\\
0&0&1&1&0&0&0&1&1&1&1&1&1\\
0&0&0&1&1&0&1&0&1&1&1&1&1\\
0&0&0&0&1&1&1&1&0&1&1&1&1\\
0&0&1&0&0&1&1&1&1&0&1&1&1\\
0&1&1&1&1&1&1&1&1&1&0&1&1\\
1&0&1&1&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&1&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&1&0\\
0&0&0&0&0&0&0&1&1&1&1&1&0\\
0&0&0&0&0&0&1&0&1&1&1&1&0\\
0&0&0&0&0&1&0&1&1&1&1&1&1\\
0&0&0&0&1&0&1&0&1&1&1&1&1\\
1&0&0&1&1&1&1&1&0&1&1&1&1\\
0&1&1&0&1&1&1&1&1&0&1&1&1\\
0&1&1&0&1&1&1&1&1&1&0&1&1\\
1&0&0&1&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&0&1&0\\
0&0&0&0&0&0&0&0&1&0&1&0&0\\
0&0&0&0&0&0&0&1&1&0&0&1&0\\
0&0&0&0&0&0&1&0&0&1&1&0&0\\
0&0&0&0&0&0&0&1&0&1&1&1&0\\
0&0&0&0&0&0&1&0&1&0&1&1&0\\
0&0&0&1&0&1&0&1&1&1&1&1&1\\
0&0&1&0&1&0&1&0&1&1&1&1&1\\
0&1&1&0&0&1&1&1&0&1&1&1&1\\
1&0&0&1&1&0&1&1&1&0&1&1&1\\
0&1&0&1&1&1&1&1&1&1&0&1&1\\
1&0&1&0&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&1&0\\
0&0&0&0&0&0&0&1&1&0&0&1&0\\
0&0&0&0&0&0&1&0&0&1&1&0&0\\
0&0&0&0&0&0&0&1&1&0&1&1&0\\
0&0&0&0&0&0&1&0&0&1&1&1&0\\
0&0&0&1&0&1&0&1&1&1&1&1&1\\
0&0&1&0&1&0&1&0&1&1&1&1&1\\
0&1&1&0&1&0&1&1&0&1&1&1&1\\
1&0&0&1&0&1&1&1&1&0&1&1&1\\
1&0&0&1&1&1&1&1&1&1&0&1&1\\
0&1&1&0&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&1&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0\\
0&0&0&0&0&0&0&0&1&0&1&1&0\\
0&0&0&0&0&0&1&0&1&1&0&1&0\\
0&0&0&0&0&0&0&1&1&1&1&0&0\\
0&1&0&0&1&0&0&1&1&1&1&1&1\\
1&0&0&0&0&1&1&0&1&1&1&1&1\\
0&0&0&1&1&1&1&1&0&1&1&1&1\\
0&0&1&0&1&1&1&1&1&0&1&1&1\\
0&1&1&1&0&1&1&1&1&1&0&1&1\\
1&0&1&1&1&0&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
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\end{pmatrix}$$

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\end{pmatrix}$$

$$\begin{pmatrix}
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0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{pmatrix}$$

Remark : By using Theorem4.16 the catalogue of s.c. chordal graphs with 4n+1 vertices can be compiled from the catalogue of s.c. chordal graphs with 4n vertices where $n > 3$ as we have done for the case of s.c. chordal graphs with 13 vertices. We note that the complexity of the algorithm which constructs all non-isomorphic s.c. chordal graphs with 4n+1 vertices from all non-isomorphic s.c. chordal graphs with 4n vertices by using the method followed above takes $O(n\zeta_{4n})$ where ζ_{4n} is the number of non-isomorphic s.c. chordal graphs with 4n vertices.

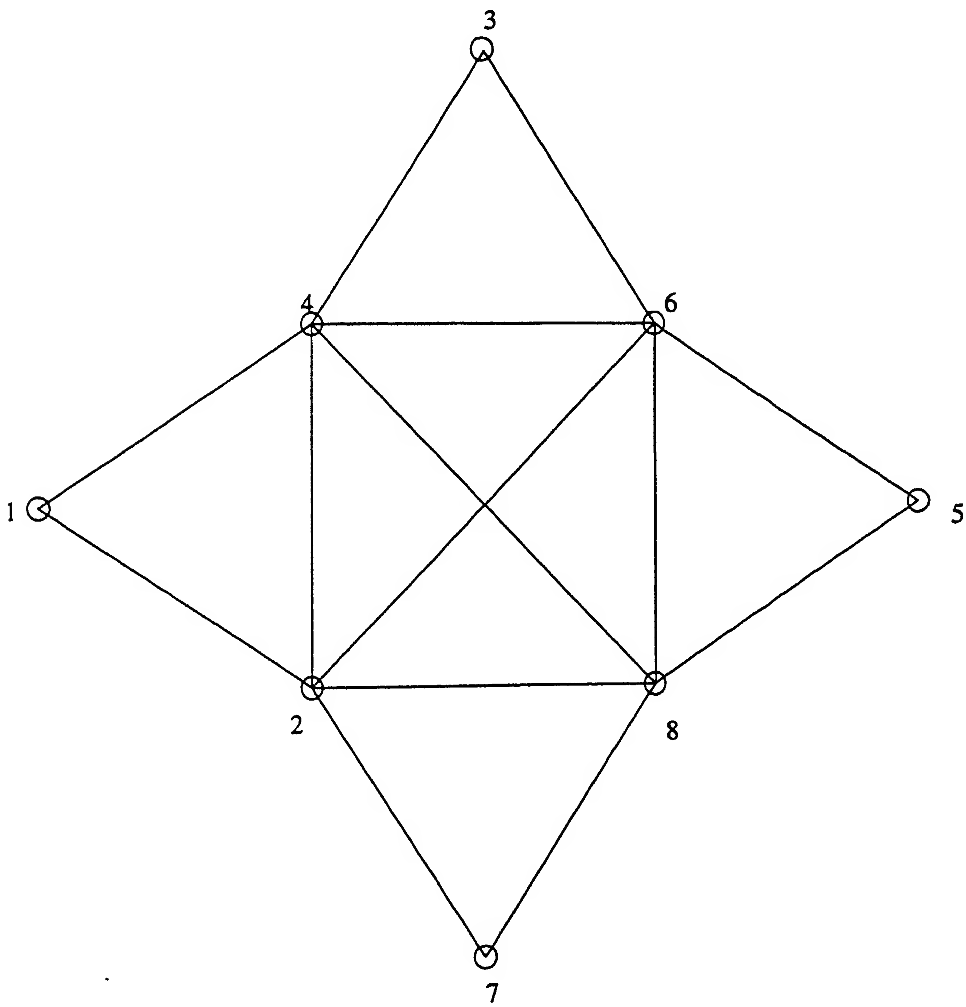

Figure 4.1a

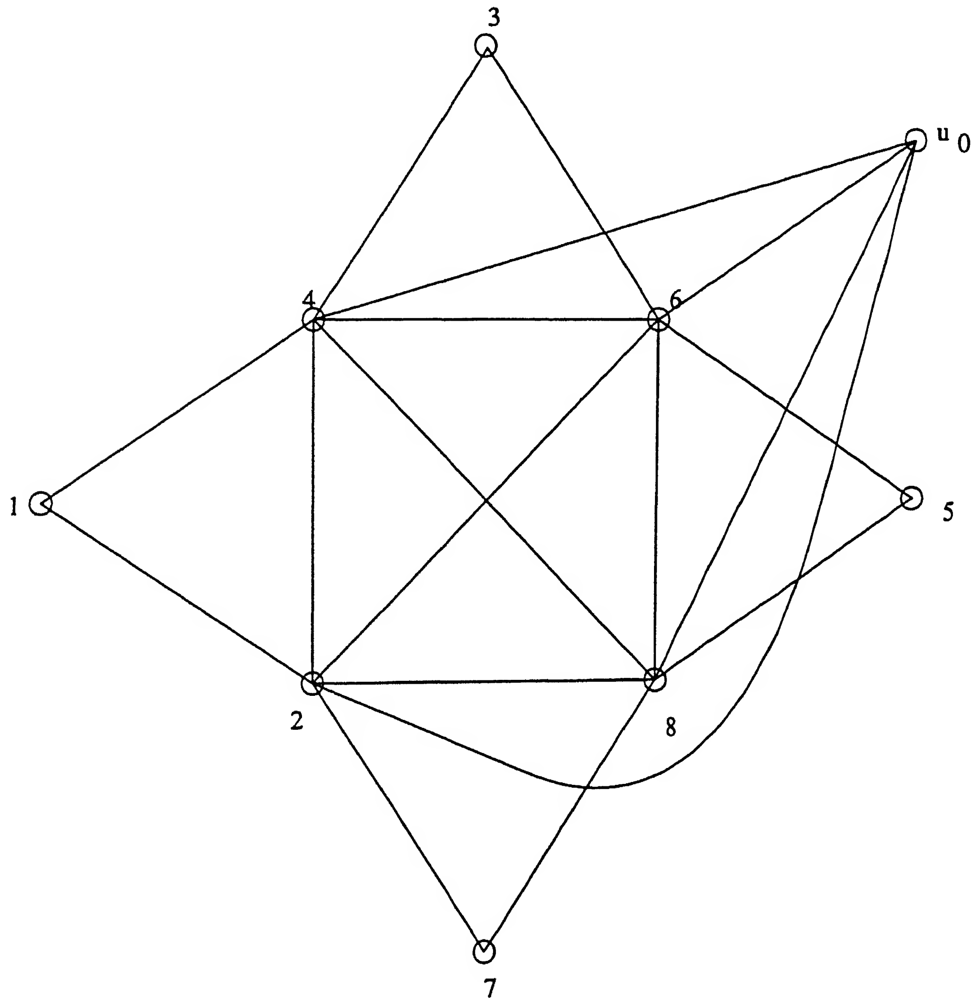

Figure 4.1b

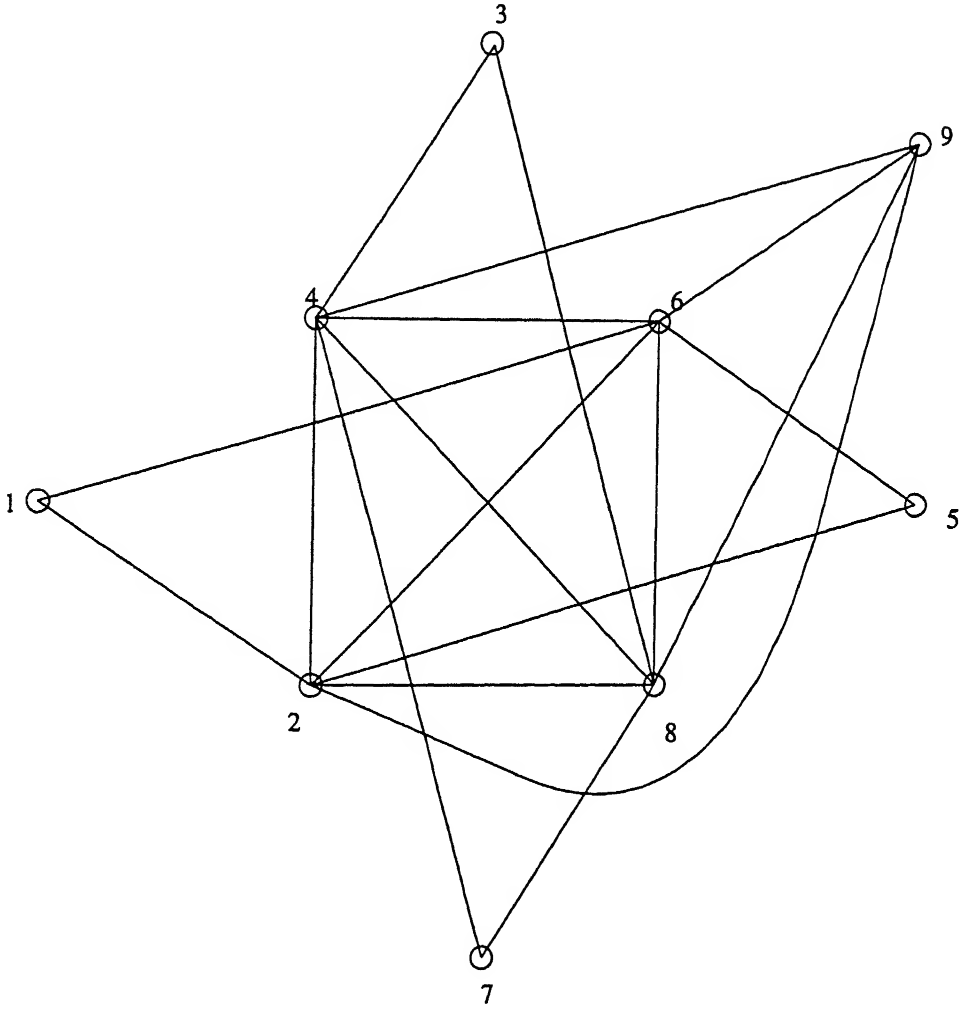

Figure 4.2a

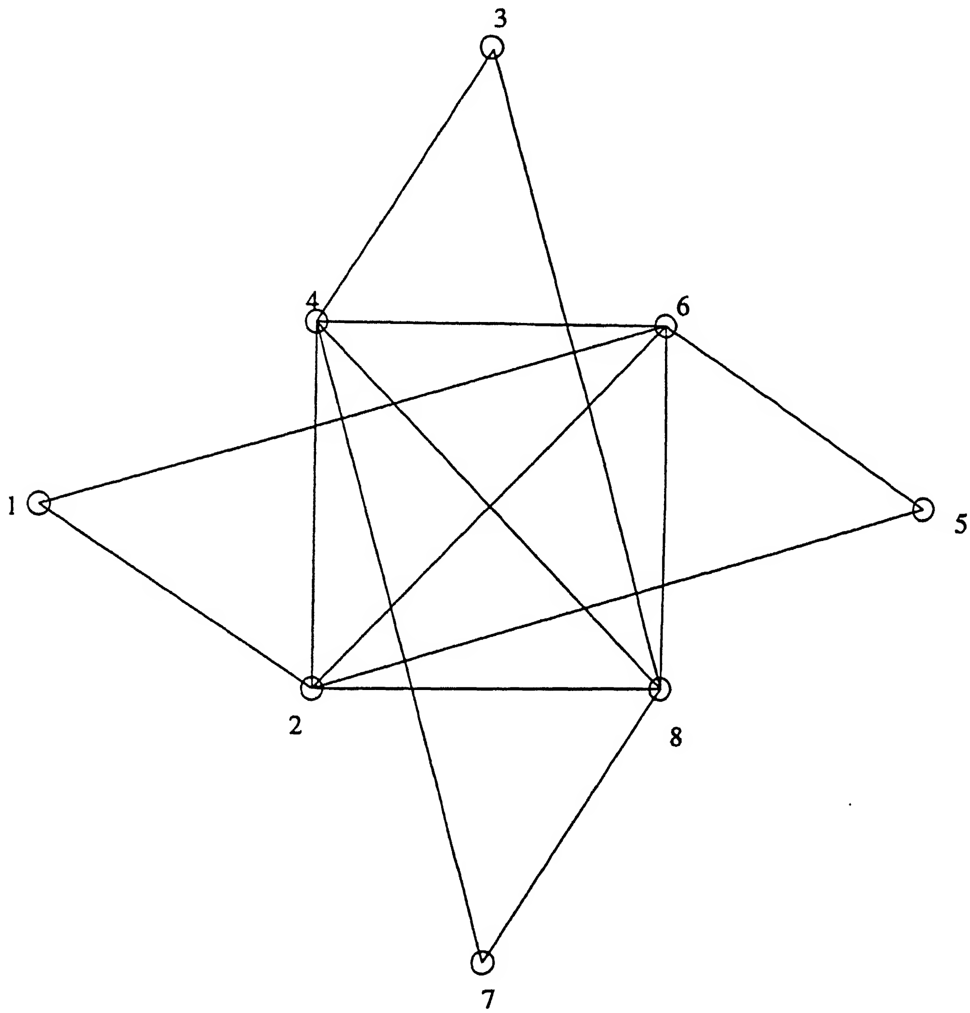

Figure 4.2b

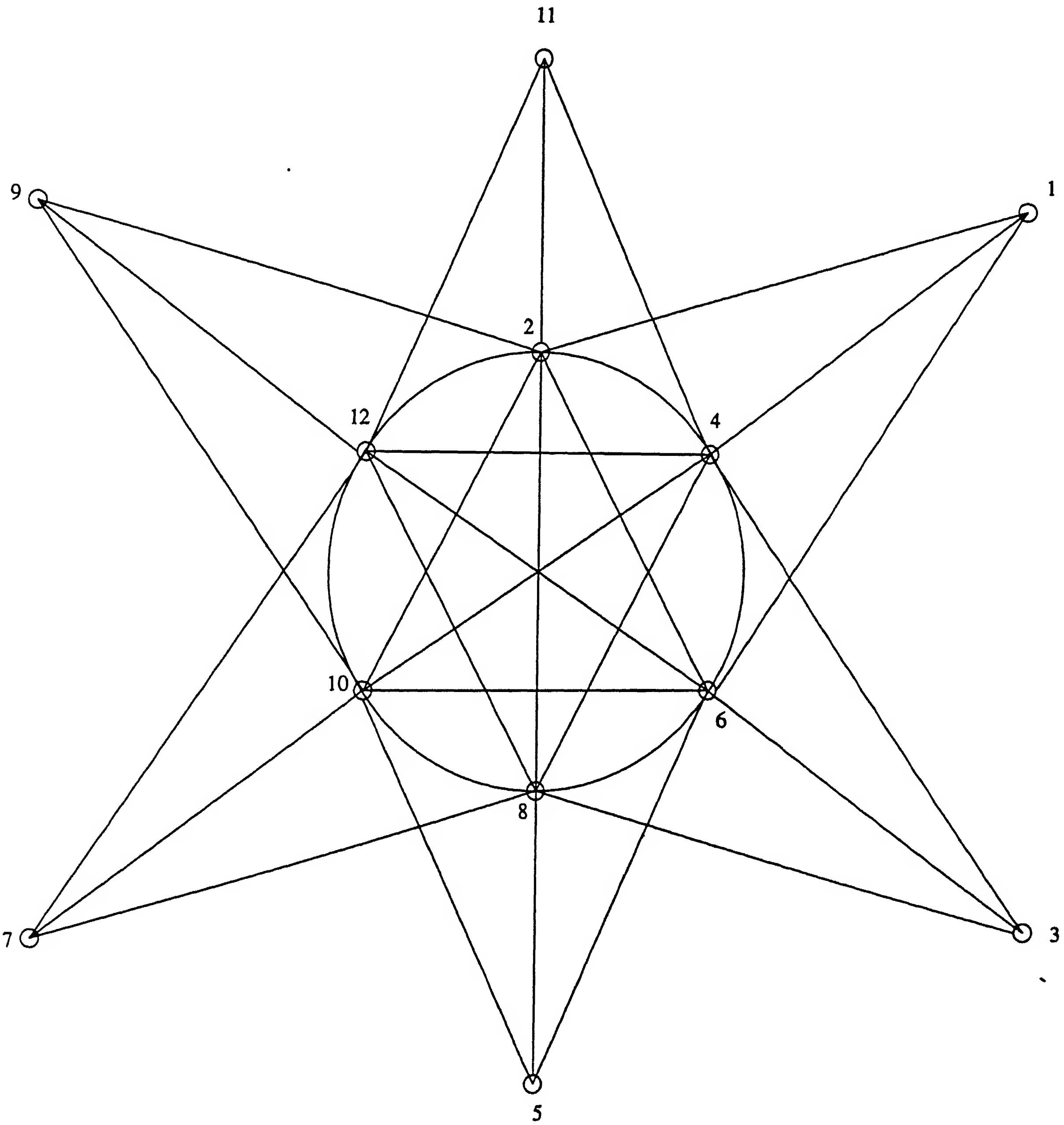

Figure 4.3a

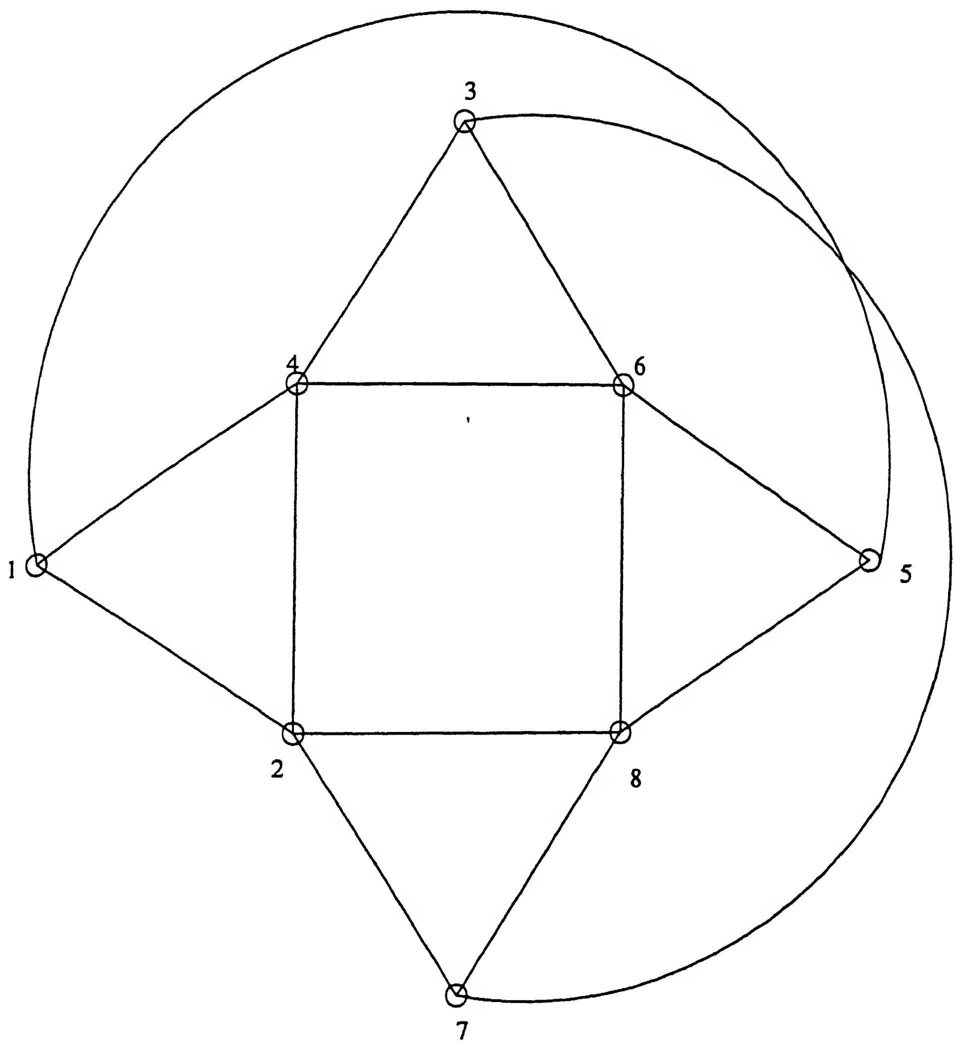

Figure 4.3b

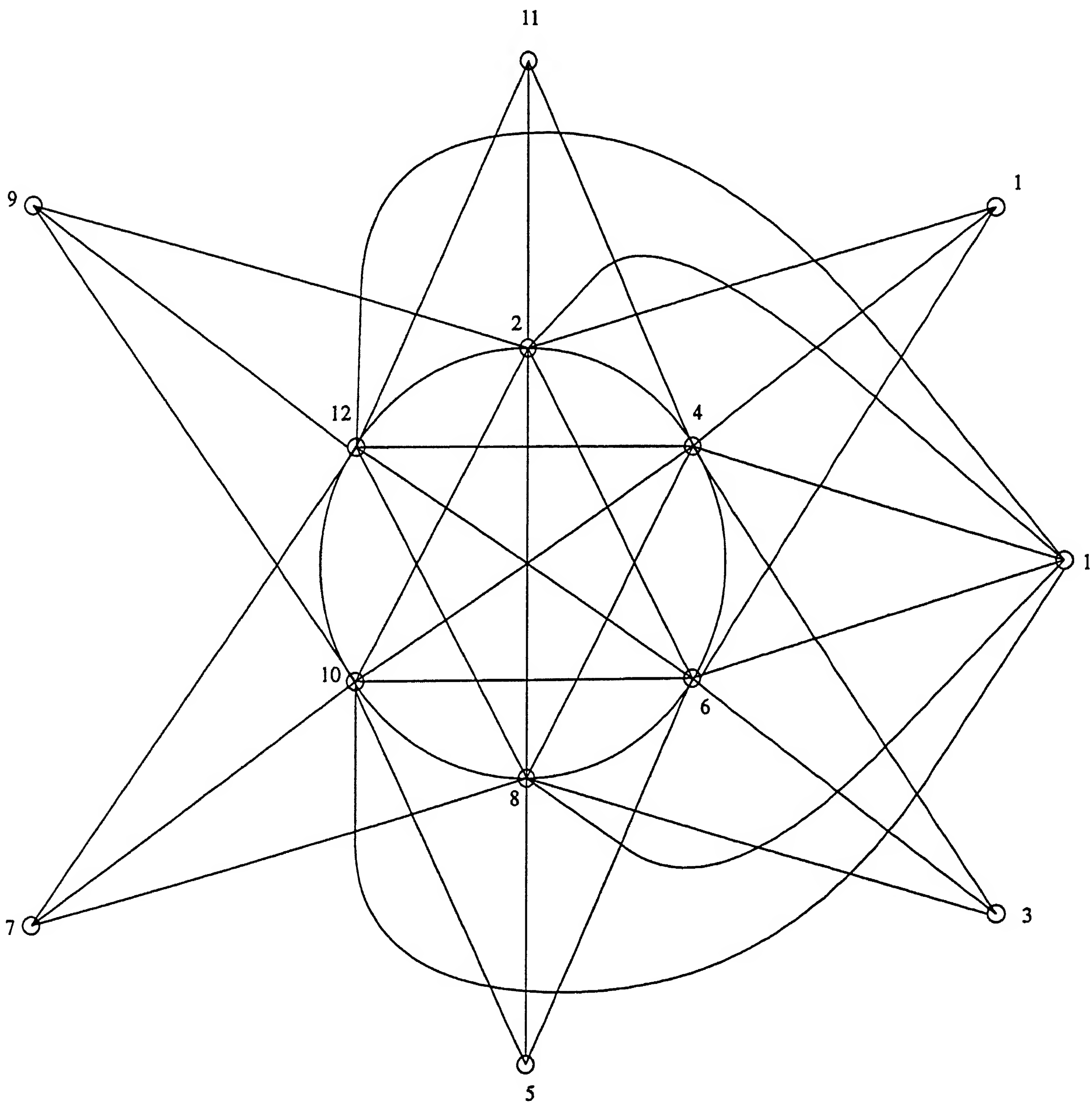

Figure 4.4a

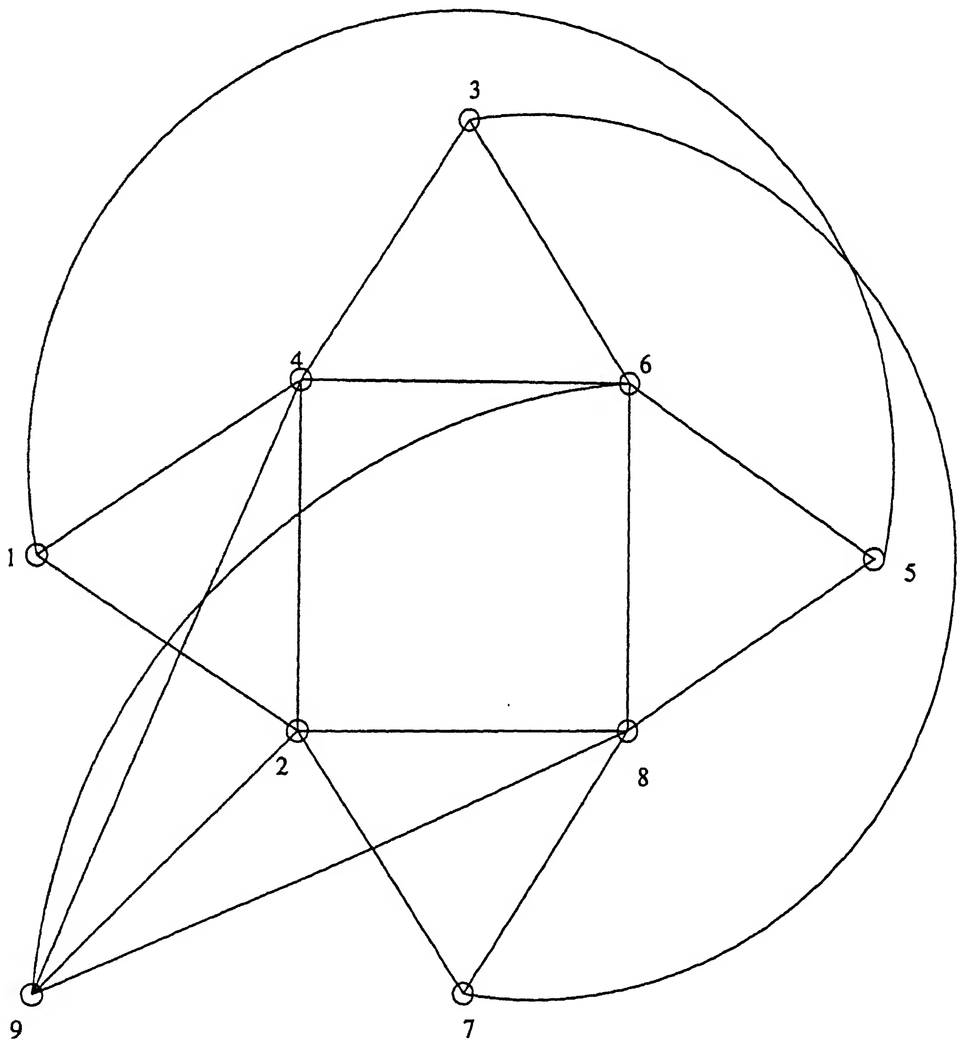

Figure 4.4b

Chapter 5

Self-complementary chordal graphs and some graph parameters

5.1 Introduction

Graph parameters have been studied because of the usefulness in determining the structure of the graph.

In this chapter we study about the following graph parameters for s.c. chordal graphs.

(a) Chromatic number.

(b) Chromatic index.

(c) Domination number.

(d) Spectrum.

In general obtaining the exact value of the chromatic number of a graph is quite difficult. However researchers had obtained bounds for the chromatic number of graphs and several classes of graphs (see [122] and [217]). In Section 5.2 we prove that the chromatic number of s.c. chordal graphs with 4n vertices is 2n and that of s.c. chordal graphs with 4n+1 vertices is 2n+1. We also obtain bounds on the chromatic number of s.c. perfect graphs and prove that the upper bounds are attained if and only if the graph is s.c. chordal.

Vizing [226] proved that the chromatic index of a graph G is $\Delta(G)$ or $\Delta(G)+1$. Since then research has progressed much in classifying the graphs with chromatic index $\Delta(G)$ (Class 1 graphs) and the graphs with chromatic index $\Delta(G)+1$ (Class 2 graphs) [81], [82] and [83]. In Section 5.3 we give a sufficient condition for a s.c. chordal graph with 4n vertices to be a Class 1 graph and establish the Class 1 property for some classes of s.c. chordal graphs with 4n vertices. We also obtain bounds for the chromatic index of s.c. chordal graphs.

Results on the domination number of graphs and several classes of graphs have been surveyed in [31], [62], [116] and [117]. Booth in [31] has proved that computing domination number of chordal graphs is NP-Complete. However linear time algorithm exists for computing independent domination number of chordal graphs [79]. In Section 5.4 we prove that the domination number and the independent domination number can be atmost 2n for s.c. chordal graphs with 4n vertices and atmost 2n+1 for s.c. chordal graphs with 4n+1 vertices.

The spectrum of a graph provides a wealth of information about the graph though by no means it specifies the graph uniquely [193]. For further reading on this topic we refer to [72] and [193]. Existence of cospectral graphs with several restrictions had been studied by researchers [72] and [193]. In Section 5.5 we prove that the least positive integer for which there exists cospectral s.c. chordal graphs is 12. We also obtain bounds for the maximum eigen-value of a s.c. chordal graph.

5.2 On the chromatic number of self-complementary chordal graphs

The following result on the chromatic number of a graph and its complement was obtained by Nordhaus and Gaddum [145].

Theorem 5.1 *(Nordhaus and Gaddum [145]) : For any graph G with p vertices the following holds.*

(i) $\lceil 2\sqrt{p}\rceil \leq \chi(G)+\chi(\bar{G}) \leq p+1$

(ii) $p \leq \chi(G)\chi(\bar{G}) \leq \lfloor(\frac{p+1}{2})^2\rfloor$.

The following result gives bounds for the chromatic number of a s.c. graph.

Theorem 5.2 : *Let G be a s.c. graph with p vertices. Then*

(i) $\lceil\sqrt{4n}\rceil \leq \chi(G) \leq 2n$ *when p=4n*

(ii) $\lceil\sqrt{4n+1}\rceil \leq \chi(G) \leq 2n+1$ *when p=4n+1.*

Proof: (i) Let G be a s.c. graph with 4n vertices. We note that $\chi(G) = \chi(\bar{G})$ since G is a s.c. graph. So by Theorem5.1 $\chi(G)$ should satisfy the inequations given by (5.1).

$$\left.\begin{array}{r}\lceil 2\sqrt{4n}\rceil \leq 2\chi(G) \leq 4n+1 \\ 4n \leq (\chi(G))^2 \leq \lfloor(\frac{4n+1}{2})^2\rfloor\end{array}\right\} \tag{5.1}$$

Then if follows that

$$\lceil\sqrt{4n}\rceil \leq \chi(G) \leq 2n .$$

(ii) Let G be a s.c. graph with 4n+1 vertices. We note that $\chi(G) = \chi(\bar{G})$ since G is a s.c. graph. So by Theorem5.1 $\chi(G)$ should satisfy the inequations given by (5.2).

$$\left.\begin{array}{r}\lceil 2\sqrt{4n+1}\rceil \leq 2\chi(G) \leq 4n+2 \\ 4n+1 \leq (\chi(G))^2 \leq \lfloor(\frac{4n+2}{2})^2\rfloor\end{array}\right\} \tag{5.2}$$

Then it follows that

$$\lceil\sqrt{4n+1}\rceil \leq \chi(G) \leq 2n+1 . \quad \square$$

A characterisation for a s.c. perfect graph to be a chordal graph in terms of its chromatic number follows.

Theorem 5.3 : *Let G be a s.c. perfect graph. Then G is chordal if and only if*

(i) $\chi(G) = 2n$ *when p=4n*

(ii) $\chi(G) = 2n+1$ *when p=4n+1.*

Proof: Let G be a chordal graph. By Theorem2.20 $\omega(G) = 2n$ when p=4n and $\omega(G) = 2n{+}1$ when p=4n+1. By the definition of a perfect graph $\chi(G) = \omega(G)$. So $\chi(G) = 2n$ when p=4n and $\chi(G) = 2n + 1$ when p=4n+1.

Let $\chi(G) = 2n$ when p=4n and $\chi(G) = 2n + 1$ when p=4n+1. By the definition of a perfect graph $\omega(G) = \chi(G)$. So $\omega(G) = 2n$ when p=4n and $\omega(G) = 2n + 1$ when p=4n+1. Then by Theorem2.20 it follows that G is a chordal graph. □

Next result gives bounds for the chromatic number of a s.c. perfect graph. The upper bounds given by this result is attained only for s.c. chordal graphs and every s.c. chordal graph attains the upper bound.

Theorem 5.4 : *Let G be a s.c. perfect graph. Then*

(i) $\lceil\sqrt{4n}\rceil \leq \chi(G) \leq 2n$ *when p=4n*

(ii) $\lceil\sqrt{4n+1}\rceil \leq \chi(G) \leq 2n + 1$ *when p=4n+1.*

Moreover, the upper bounds given by (i) and (ii) are attained if and only if G is a chordal graph.

Proof: Follows from Theorem5.2 and Theorem5.3. □

5.3 On the chromatic index of self-complementary chordal graphs

The following result was independently obtained by Bezhad et. al.[28] and Vizing [227].

Theorem 5.5 *(Behzad et. al. [28] and Vizing [227]) : The complete graph K_{2n} where n is a positive integer is a Class 1 graph.*

A sufficient condition for a s.c. chordal graph with 4n vertices to be a Class 1 graph is given by the following Theorem.

Theorem 5.6 : *Let G be a s.c. chordal graph with p=4n. Let $\sigma^* = \sigma_1^*\sigma_2^*\cdots\sigma_s^*$ where $\sigma_i^* = (v_{i1}v_{i2}\cdots v_{ip_i})$ for all $1 \leq i \leq s$ be a star c.p. of G. Let H be the graph $< T^* >$ where*

$T^* = [Even(\sigma^*), Odd(\sigma^*)]$. *Then G is a Class 1 graph if both (i) and (ii) are satisfied.*

(i) *H is a Class 1 graph.*

(ii) *There exists a vertex* v_{ij} *of G such that* $deg_G(v_{ij}) = \Delta(G)$ *and* $deg_H(v_{ij}) = \Delta(H)$.

Proof: Let H be a Class 1 graph. Let v_{ij} be a vertex of G such that $deg_G(v_{ij}) = \Delta(G)$ and $deg_H(v_{ij}) = \Delta(H)$. Let H' be the graph $< Even(\sigma^*) >$. By Theorem4.1 $v_{ij} \in Even(\sigma^*)$. The subgraph H is a Class 1 graph implies that the edges of H can be properly colored with the colors $a_1, a_2, \ldots, a_{\Delta(H)}$. By Theorem3.8 $H' \cong K_{2n}$ since $|Even(\sigma^*)| = 2n$. So $Odd(\sigma^*)$ is a stable set of G since $Odd(\sigma^*) = \sigma^*(Even(\sigma^*))$. By Theorem5.5 the graph H' is a Class 1 graph since $H' \cong K_{2n}$. Hence the edges of H' can be properly colored with the colors $b_1, b_2, \ldots, b_{2n-1}$. Every edge of G is either an edge of the graph H or an edge of the graph H' since $Odd(\sigma^*)$ is a stable set of G. So the edges of the graph G can be properly colored with the colors $a_1, a_2, \ldots, a_{\Delta(H)}, b_1, b_2, \ldots, b_{2n-1}$. The vertex $v_{ij} \in Even(\sigma^*)$ implies $deg_{H'}(v_{ij}) = 2n - 1$. Hence $\Delta(G) = deg_G(v_{ij}) = deg_H(v_{ij}) + deg_{H'}(v_{ij}) = \Delta(H) + 2n - 1$ since every edge incident with v_{ij} is either an edge of H or an edge of H'. Then it follows that G is a Class 1 graph since the edges of G can be properly colored with $\Delta(G)$ colors $a_1, a_2, \ldots, a_{\Delta(H)}, b_1, b_2, \ldots, b_{2n-1}$. □

The following result is due to König [125].

Theorem 5.7 *(König [125]) : Every bipartite graph is a Class 1 graph.*

A sufficient condition for a s.c. chordal graph with 4n vertices to be a Class 1 graph in terms of its induced subgraph is given by the following result.

Theorem 5.8 *: Let G be a s.c. chordal graph with p=4n. Let* σ^* *be a star c.p. of G. Then G is a Class 1 graph if* $< T^* >$ *is a regular graph where* $T^* = [Even(\sigma^*), Odd(\sigma^*)]$.

Proof: Let $\sigma^* = \sigma_1^* \sigma_2^* \cdots \sigma_s^*$ where $\sigma_i^* = (v_{i1} v_{i2} \cdots v_{ip_i})$ for $1 \leq i \leq s$. Let H be the graph $< T^* >$. The graph H is regular implies that $deg_H(v_{ij}) = \Delta(H)$ for all the vertices v_{ij} of G. So there exists a vertex v_{ij} of G such that $deg_G(v_{ij}) = \Delta(G)$ and $deg_H(v_{ij}) = \Delta(H)$ since V(G)=V(H). We note that H is a bipartite graph since the sets $Even(\sigma^*)$ and $Odd(\sigma^*)$ partition V(H) such that every edge of H is incident with a vertex of $Even(\sigma^*)$ and a vertex

of $Odd(\sigma^*)$. By Theorem5.7 H is a Class 1 graph. Therefore by Theorem5.6 the result follows. □

Lemma 5.1 : *Let G be a s.c. graph with $4n$ vertices and a c.p. $\sigma = (1\ 2 \cdots 4n)$. Then $< T >$ is a regular graph where $T = [Even(\sigma), Odd(\sigma)]$.*

Proof: Let H be the graph $< T >$. By Lemma2.2 $[j, \sigma^{2i-1}(j)] \in E(G)$ if and only if $[\sigma^{4n-(2i-1)}(j), \sigma^{4n}(j)] = [j, \sigma^{4n-(2i-1)}(j)] \notin E(G)$ for all $1 \leq i \leq n$ and $1 \leq j \leq 4n$. Hence $deg_H(i) = n$ for all the vertices i of G such that $1 \leq i \leq 4n$. □

The following Theorem establishes the Class 1 property for a class of s.c. chordal graphs with 4n vertices.

Theorem 5.9 : *Let G be a s.c. chordal graph with $4n$ vertices. Let $\sigma^* = (1\ 2 \cdots 4n)$ be a star c.p. of G. Then G is a Class 1 graph.*

Proof: Follows from Theorem5.8 and Lemma5.1.□

Alavi et. al. [4] and Vizing [227] independently obtained the following result.

Theorem 5.10 *(Alavi et. al. [4] and Vizing [227]) : Let G be a graph with p vertices. Then*

(i) $2\lfloor\frac{(p+1)}{2}\rfloor - 1 \leq \chi'(G) + \chi'(\bar{G}) \leq p + 2\lfloor\frac{(p-2)}{2}\rfloor$

(ii) $0 \leq \chi'(G)\chi'(\bar{G}) \leq (p-1)(2\lfloor\frac{p}{2}\rfloor - 1)$.

The following result is on bounds for the chromatic index of a s.c. graph.

Theorem 5.11 : *Let G be a s.c. graph. Then*

(i) $2n \leq \chi'(G) \leq 4n - 1$ *when p=4n*

(ii) $2n + 1 \leq \chi'(G) \leq 4n - 1$ *when p=4n+1.*

Proof: Let p=4n. We note that $\chi'(G) = \chi'(\bar{G})$ since G is a s.c. graph. By Theorem5.10 $\chi'(G)$ should satisfy the inequations given by (5.3).

$$\left.\begin{array}{l} 2\lfloor\frac{(4n+1)}{2}\rfloor - 1 \leq 2\chi'(G) \leq 4n + 2\lfloor\frac{(4n-2)}{2}\rfloor \\ 0 \leq (\chi'(G))^2 \leq (4n-1)(2\lfloor\frac{4n}{2}\rfloor - 1) \end{array}\right\} \qquad (5.3)$$

Hence

$$2n \leq \chi'(G) \leq 4n - 1.$$

(ii) Let p=4n+1. The graph G is a s.c. graph implies that $\chi'(G) = \chi'(\bar{G})$. By Theorem5.10 $\chi'(G)$ should satisfy the inequations given by (5.4).

$$\left.\begin{array}{l} 2\lfloor\frac{(4n+2)}{2}\rfloor - 1 \leq 2\chi'(G) \leq 4n + 1 + 2\lfloor\frac{(4n-1)}{2}\rfloor \\ 0 \leq (\chi'(G))^2 \leq 4n(2\lfloor\frac{(4n+1)}{2}\rfloor - 1) \end{array}\right\} \quad (5.4)$$

Therefore

$$2n + 1 \leq \chi'(G) \leq 4n - 1. \quad \square$$

Now we obtain bounds for the chromatic index of a s.c. chordal graph.

Theorem 5.12 : *Let G be a s.c. chordal graph. Then*

(i) $2n \leq \chi'(G) \leq 4n - 1$ *when p=4n*

(ii) $2n + 1 \leq \chi'(G) \leq 4n - 1$ *when p=4n+1.*

Proof: Follows from Theorem5.11. □

5.4 On the domination number of self-complementary chordal graphs

For a s.c. chordal graph with a star c.p. σ^* the set of all odd labelled vertices of σ^* is a kernel of the graph.

Theorem 5.13 : *Let G be a s.c. chordal graph. Let σ^* be a star c.p. of G. Then $Odd(\sigma^*)$ is a kernel of G.*

Proof: Casei : Let G be a s.c. chordal graph with 4n vertices. Let $\sigma^* = \sigma_1^*\sigma_2^* \cdots \sigma_s^*$ where $\sigma_i^* = (v_{i1}v_{i2}\cdots v_{ip_i})$ for $1 \leq i \leq s$. By Theorem2.1 $p_i = 4n_i$ for some n_i where $1 \leq i \leq s$. Let $v_{ij} \in Even(\sigma^*)$. By Lemma2.2 either $[v_{ij}, \sigma^*(v_{ij})] \in E(G)$ or $[\sigma^{*^{4n_i-1}}(v_{ij}), \sigma^{*^{4n_i}}(v_{ij})] = [v_{ij}, \sigma^{*^{4n_i-1}}(v_{ij})] \in E(G)$. Also $v_{ij} \in Even(\sigma^*)$ implies that both the vertices $\sigma^*(v_{ij})$ and $\sigma^{*^{4n_i-1}}(v_{ij})$ belong to $Odd(\sigma^*)$. Hence for all the vertices $v_{ij} \in Even(\sigma^*)$ there exists a

vertex $v_{lk} \in Odd(\sigma^*)$ such that $[v_{ij}, v_{lk}] \in E(G)$. By Theorem3.8 $< Even(\sigma^*) >$ is a complete subgraph of G. So the set $Odd(\sigma^*)$ is a stable set of G since $Odd(\sigma^*) = \sigma^*(Even(\sigma^*))$. Then it follows that $Odd(\sigma^*)$ is a kernel of G since the sets $Even(\sigma^*)$ and $Odd(\sigma^*)$ partition V(G).

Caseii : Let G be a s.c. chordal graph with 4n+1 vertices. Let $\sigma^* = \sigma_1^*\sigma_2^* \cdots \sigma_s^*$ where $\sigma_1^* = (v_0)$ and $\sigma_i^* = (v_{i1}v_{i2}\cdots v_{ip_i})$ for $2 \leq i \leq s$. By Theorem2.1 $p_i = 4n_i$ for some n_i where $2 \leq i \leq s$. Let $v_{ij} \in Even(\sigma^*)$. We note that v_{ij} is distinct from v_0 since $v_0 \in Odd(\sigma^*)$. By Lemma2.2 either $[v_{ij}, \sigma^*(v_{ij})] \in E(G)$ or $[\sigma^{*^{4n_i-1}}(v_{ij}), \sigma^{*^{4n_i}}(v_{ij})] = [v_{ij}, \sigma^{*^{4n_i-1}}(v_{ij})] \in E(G)$. Also $v_{ij} \in Even(\sigma^*)$ implies that both the vertices $\sigma^*(v_{ij})$ and $\sigma^{*^{4n_i-1}}(v_{ij})$ belong to $Odd(\sigma^*)$. Hence for all the vertices $v_{ij} \in Even(\sigma^*)$ there exists a vertex $v_{lk} \in Odd(\sigma^*)$ such that $[v_{ij}, v_{lk}] \in E(G)$. By Theorem3.13 $< \{v_0\} \cup Even(\sigma^*) >$ is a complete subgraph of G. So the set $Odd(\sigma^*)$ is a stable set of G since $Odd(\sigma^*) = \sigma^*(\{v_0\} \cup Even(\sigma^*))$. It follows that $Odd(\sigma^*)$ is a kernel of G since the sets $Even(\sigma^*)$ and $Odd(\sigma^*)$ partition V(G). □

The following result gives an upper bound for the independent domination number of a s.c. chordal graph.

Theorem 5.14 : *Let G be a s.c. chordal graph. Then*

(i) $\delta_i(G) \leq 2n$ *when p=4n*

(ii) $\delta_i(G) \leq 2n + 1$ *when p=4n+1.*

Proof: Let σ^* be a star c.p. of G. By Theorem5.13 $Odd(\sigma^*)$ is a kernel of G. Hence $\delta_i(G) \leq 2n$ and $\delta_i(G) \leq 2n + 1$ accordingly as p=4n and p=4n+1 respectively since $|Odd(\sigma^*)| = 2n$ when p=4n and $|Odd(\sigma^*)| = 2n + 1$ when p=4n+1. □

Lemma 5.2 : *For any graph G,* $\delta_0(G) \leq \delta_i(G)$.

Proof: Follows from the definition of $\delta_0(G)$ and $\delta_i(G)$. □

The following result gives an upper bound for the domination number of a s.c. chordal graph.

Theorem 5.15 : *Let G be a s.c. chordal graph. Then*

(i) $\delta_0(G) \leq 2n$ *when p=4n*

(ii) $\delta_0(G) \leq 2n+1$ *when p=4n+1.*

Proof: Follows from Theorem5.14 and Lemma5.2. □

5.5 On the spectrum of self-complementary chordal graphs

No two non-isomorphic s.c. chordal graph with p vertices where $p < 12$ have the same spectrum.

Theorem 5.16 : *No two s.c. chordal graph with 4 vertices are cospectral.*

Proof: Vacuously true since G has only one non-isomorphic s.c. chordal graph with 4 vertices. □

Theorem 5.17 : *No two s.c. chordal graph with 5 vertices are cospectral.*

Proof: Vacuously true since G has only one non-isomorphic s.c. chordal graph with 5 vertices. □

Theorem 5.18 : *No two s.c. chordal graph with 8 vertices are cospectral.*

Proof: Adjacency matrices of the three non-isomorphic s.c. chordal graphs with 8 vertices and their characteristic polynomial are given below.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Characteristic polynomial of the above matrix is $16\lambda^2 + 40\lambda^3 + 21\lambda^4 - 16\lambda^5 - 14\lambda^6 + \lambda^8$.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Characteristic polynomial of the above matrix is $1-4\lambda-6\lambda^2+20\lambda^3+11\lambda^4-20\lambda^5-14\lambda^6+\lambda^8$.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Characteristic polynomial of the above matrix is $-16\lambda-12\lambda^2+32\lambda^3+25\lambda^4-16\lambda^5-14\lambda^6+\lambda^8$.

We note that none of the above polynomials can be obtained from any one of the other two polynomials by multiplying by a real number. Hence no pair of non-isomophic s.c. chordal graphs with 8 vertices have the same spectrum. □

Theorem 5.19 : *No two s.c. chordal graph with 9 vertices are cospectral.*

Proof: Adjacency matrices of the three non-isomorphic s.c. chordal graphs with 9 vertices and their characteristic polynomial are given below.

$$\begin{pmatrix} 0&1&0&0&0&1&0&0&0\\ 1&0&0&1&1&1&0&1&1\\ 0&0&0&1&0&0&0&1&0\\ 0&1&1&0&0&1&1&1&1\\ 0&1&0&0&0&1&0&0&0\\ 1&1&0&1&1&0&0&1&1\\ 0&0&0&1&0&0&0&1&0\\ 0&1&1&1&0&1&1&0&1\\ 0&1&0&1&0&1&0&1&0 \end{pmatrix}$$

Characteristic polynomial of the above matrix is $32\lambda^3 + 68\lambda^4 + 25\lambda^5 - 28\lambda^6 - 18\lambda^7 + \lambda^9$.

$$\begin{pmatrix} 0&1&0&0&0&0&0&0&0\\ 1&0&0&1&1&1&1&1&1\\ 0&0&0&1&0&0&0&0&0\\ 0&1&1&0&1&1&1&1&1\\ 0&1&0&1&0&1&0&0&0\\ 0&1&0&1&1&0&0&1&1\\ 0&1&0&1&0&0&0&1&0\\ 0&1&0&1&0&1&1&0&1\\ 0&1&0&1&0&1&0&1&0 \end{pmatrix}$$

Characteristic polynomial of the above matrix is $5\lambda - 16\lambda^2 - 6\lambda^3 + 40\lambda^4 + 11\lambda^5 - 32\lambda^6 - 18\lambda^7 + \lambda^9$.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Characteristic polynomial of the above matrix is $-32\lambda^2 - 12\lambda^3 + 60\lambda^4 + 29\lambda^5 - 28\lambda^6 - 18\lambda^7 + \lambda^9$.

We note that none of the above polynomials can be obtained from any one of the other two polynomials by multiplying by a real number. Hence no pair of non-isomorphic s.c. chordal graphs with 9 vertices have the same spectrum. □

Cospectral s.c. chordal graphs with 12 vertices exists.

Theorem 5.20 : *There exists cospectral s.c. chordal graphs with 12 vertices.*

Proof: Consider the two graphs with 12 vertices shown in Figure 5.1 and Figure 5.2. They are s.c. graphs since (1 11 2 12)(6 9 5 8 4 7 3 10) and (1 12 2 11)(6 7 5 8)(3 10 4 9) are c.p.'s of the graphs in Figure 5.1 and Figure 5.2 respectively. By Theorem2.20 both these graphs are chordal since their clique number is 6. These two graphs are non-isomorphic since the graph in Figure 5.1 has a vertex of degree 1 whereas the graph in Figure 5.2 does not have a vertex of degree 1.

The adjacency matrix and the characteristic polynomial of the s.c. chordal graph shown in Figure 5.1 follows.

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0&0&1&1&1\\ 0&0&0&0&0&0&1&1&0&0&1&1\\ 0&0&0&0&0&0&0&1&1&0&1&1\\ 0&0&0&0&0&0&0&1&1&1&1&1\\ 0&0&1&1&0&0&0&1&1&1&1&1\\ 0&0&0&1&1&0&1&0&1&1&1&1\\ 0&0&0&0&1&1&1&1&0&1&1&1\\ 0&0&1&0&0&1&1&1&1&0&1&1\\ 0&1&1&1&1&1&1&1&1&1&0&1\\ 1&0&1&1&1&1&1&1&1&1&1&0 \end{pmatrix}$$

The characteristic polynomial of the above matrix is $-16\lambda+20\lambda^2+112\lambda^3-91\lambda^4-256\lambda^5+83\lambda^6+248\lambda^7+20\lambda^8-88\lambda^9-33\lambda^{10}+\lambda^{12}$.

The adjacency matrix and the characteristic polynomial of the s.c. chordal graph shown in Figure 5.2 follows.

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&1&0&0&1\\ 0&0&0&0&0&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&0&0&1&1&1&1&1\\ 0&0&0&0&0&0&1&0&1&1&1&1\\ 0&0&0&0&0&1&0&1&1&1&1&1\\ 0&0&0&0&1&0&1&0&1&1&1&1\\ 1&0&0&1&1&1&1&1&0&1&1&1\\ 0&1&1&0&1&1&1&1&1&0&1&1\\ 0&1&0&1&1&1&1&1&1&1&0&1\\ 1&0&1&0&1&1&1&1&1&1&1&0 \end{pmatrix}$$

The characteristic polynomial of the above matrix is $-16\lambda+20\lambda^2+112\lambda^3-91\lambda^4-256\lambda^5+83\lambda^6+248\lambda^7+20\lambda^8-88\lambda^9-33\lambda^{10}+\lambda^{12}$.

We note that the characteristic polynomial of the s.c. chordal graphs with 12 vertices shown in Figure 5.1 and Figure 5.2 are the same. Therefore they have the same spectrum.□

The following result is a consolidation of the above results.

Theorem 5.21 : *The smallest positive integer for which there exist cospectral s.c. chordal graphs is 12.*

Proof: Follows from Theorem5.16, Theorem5.17, Theorem5.18, Theorem5.19 and Theorem5.20. □

Remark: In the above results characteristic polynomials of the adjacency matrices of s.c. chordal graphs were computed using the package 'Mathematica'. For more details about this package we refer to [232].

The following result gives bounds for the maximum eigenvalue of a graph.

Theorem 5.22 *(Schwenk and Wilson [193]) : Let G be a connected graph. Then either (i) or (ii) holds.*

(i) $\frac{2q}{p} < \lambda_{max}(G) < \Delta(G)$

(ii) $\frac{2q}{p} = \lambda_{max}(G) = \Delta(G)$, *G is a regular graph and* $(1, 1, \ldots, 1)$ *is an eigen vector of G.*

Lemma 5.3 : *Let G be a s.c. chordal graph. Then G is not regular.*

Proof: Follows from Theorem4.1 and Theorem4.2. □

Lemma 5.4 : *Let G be a s.c. graph. Then G is connected.*

Proof: For every graph either the graph or its complement is connected. Hence G is connected since G is a s.c. graph. □

Lemma 5.5 : *Let G be a s.c. graph with p vertices. Then* $q = \frac{p(p-1)}{4}$.

Proof: The graph K_p has $\frac{p(p-1)}{2}$ edges. The graph G is s.c. implies it has exactly half the number of edges of K_p. Hence G has $\frac{p(p-1)}{4}$ edges. □

Bounds for the maximum eigen value of a s.c. chordal graph are given by the following result.

Theorem 5.23 : *Let G be a s.c. chordal graph. Then*

(i) $\frac{(4n-1)}{2} < \lambda_{max}(G) < \Delta(G)$ *when p=4n*

(ii) $2n < \lambda_{max}(G) < \Delta(G)$ *when p=4n+1.*

Proof: Let G be a s.c. chordal graph. By Lemma5.3 and Lemma5.4 G is connected and not regular.Then by Theorem5.22 and Lemma5.5 the result follows. □

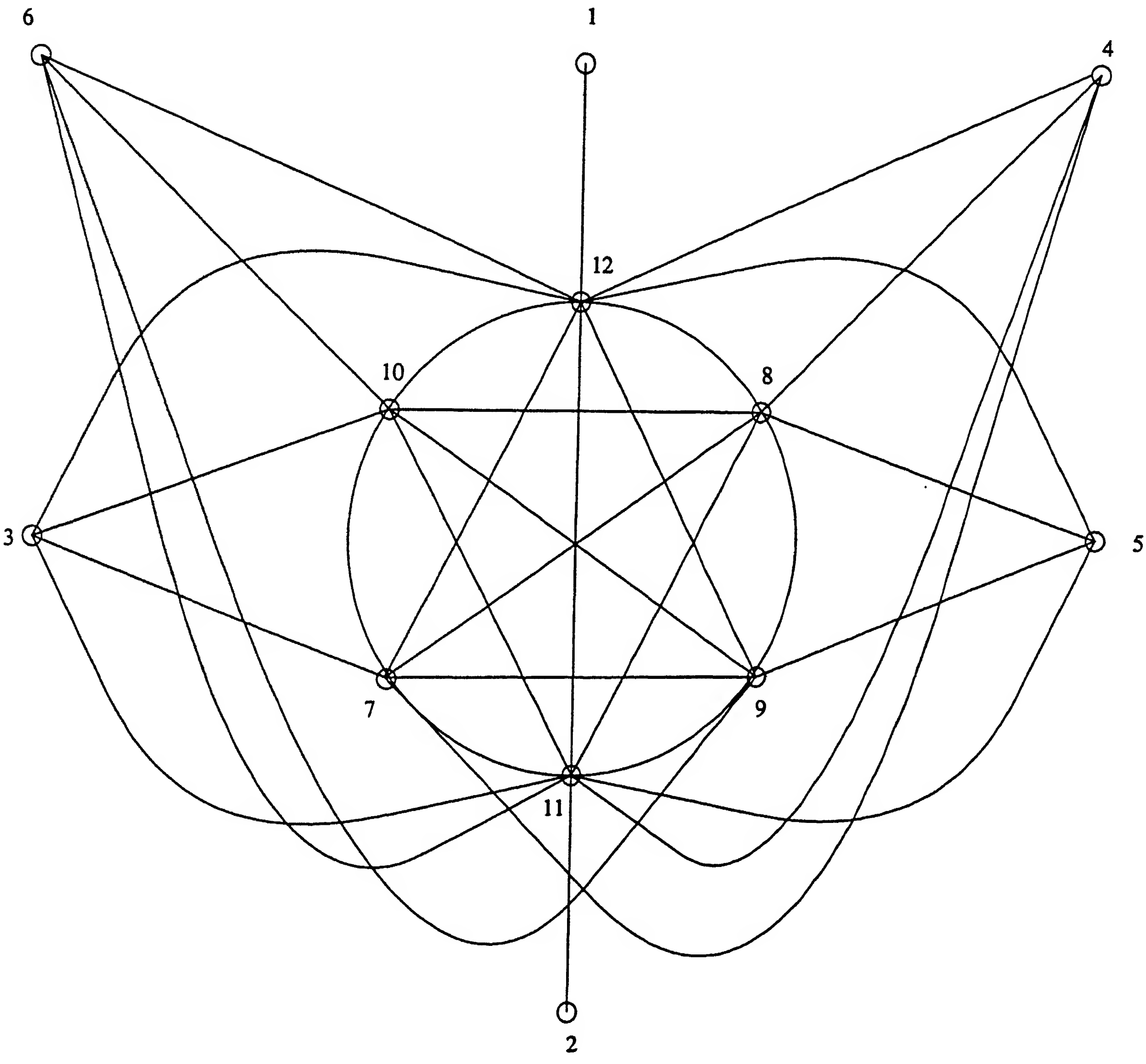

Figure 5.1

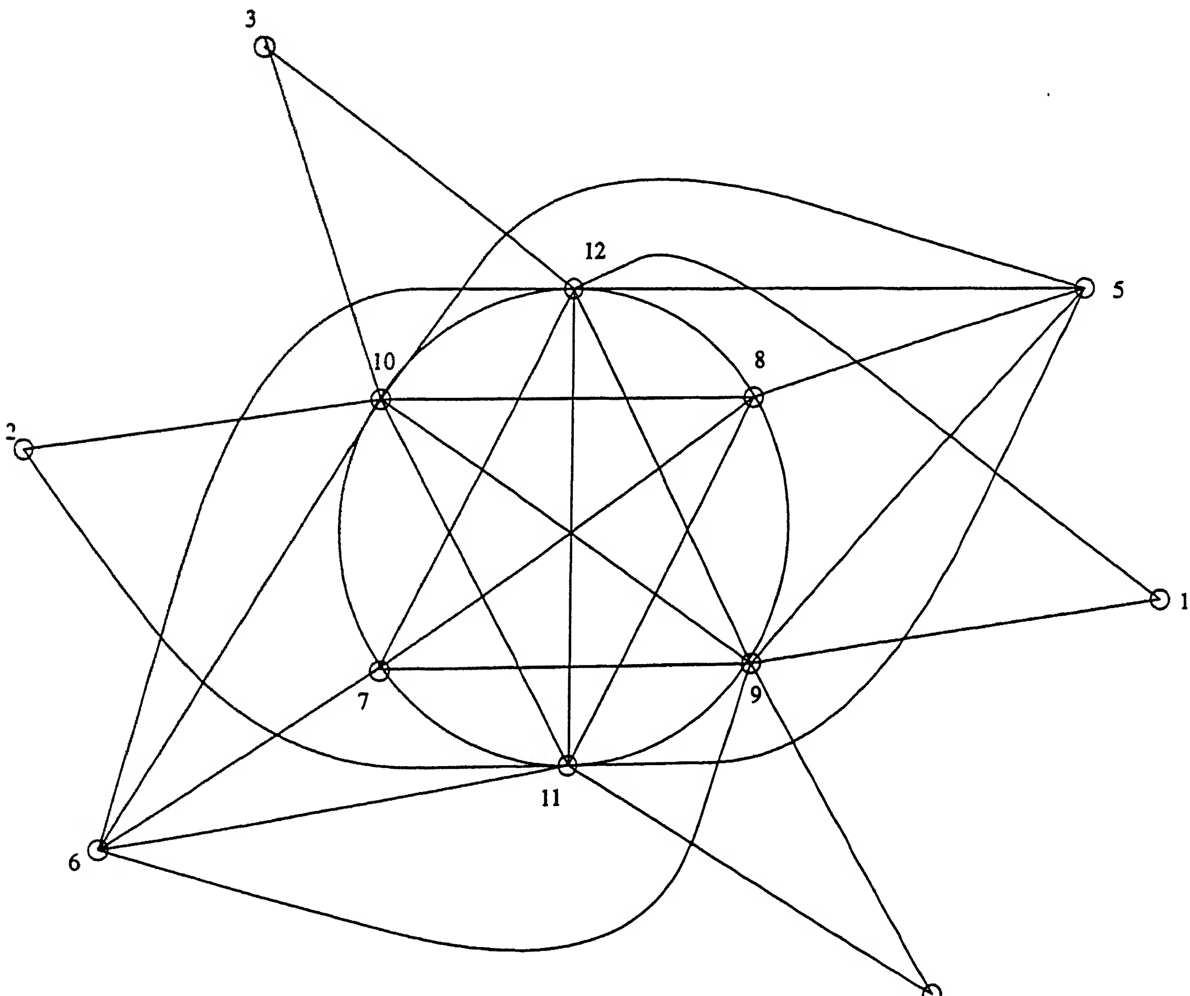

Figure 5.2

Chapter 6

Conclusion

In this Chapter we pose some research problems on s.c. chordal graphs.

In Chapter4 we proved that the isomorphism of s.c. chordal graphs is polynomially equivalent to the isomorphism of s.c. chordal graphs with 4n vertices and the isomorphism of s.c. chordal graphs with 4n+1 vertices. We conjecture the following on the algorithmic complexity of the isomorphism of s.c. chordal graphs.

Conjecture 6.1 : *The isomorphism of s.c. chordal graphs is polynomially equivalent to the general graph isomorphism (the isomorphism of graphs).*

By Theorem4.11, Conjecture6.1 could be equivalently posed as follows.

Conjecture 6.2 : *The isomorphism of s.c. chordal graphs with 4n vertices (or 4n+1 vertices) is polynomially equivalent to the general graph isomorphism (the isomorphism of graphs).*

In Chapter 4 we proved that recognising whether a given s.c. graph is chordal or not takes linear time. However, the problem of recognising whether a given graph is s.c. chordal or not (the recognition problem of s.c. chordal graphs) seems to be algorithmically as hard as that of the general graph isomorphism. So we pose the following Conjecture.

Conjecture 6.3 : *The recognition problem of s.c. chordal graphs is polynomially equivalent to the general graph isomorphism (the isomorphism of graphs).*

From Theorem4.16 it follows that the number of non-isomorphic s.c. chordal graphs with 4n vertices is equal to the number of non-isomorphic s.c. chordal graphs with 4n+1 vertices. However the problem of counting the number of non-isomorphic s.c. chordal graphs with 4n vertices or 4n+1 vertices for each positive integer n seems to be quite difficult. Hence we pose this as an open problem.

Problem 6.1 : *Count the number of non-isomorphic s.c. chordal graphs with 4n vertices for each positive integer n .*

Equivalently the above problem could be posed as follows.

Problem 6.2 : *Count the number of non-isomorphic s.c. chordal graphs with 4n+1 vertices for each positive integer n .*

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